TRIGONOMETRIC FUNCTIONS

HAZIQ SYAZWAN BIN SAJALI

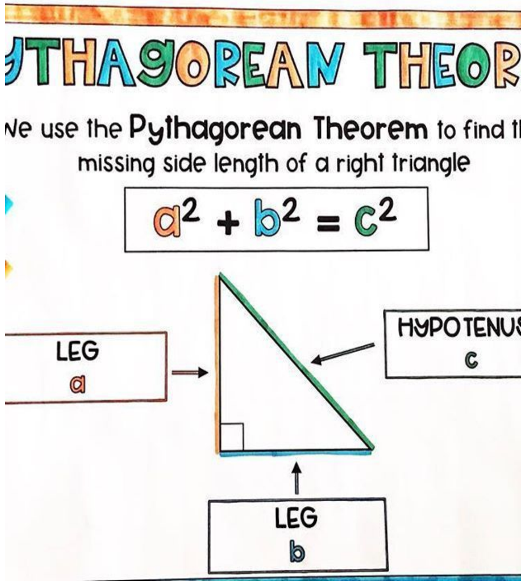
TRIGONOMETRIC FORMULAE

1. $sin^2 A + cos^2 A = 1$
2. $sec^2 A = 1 + tan^2 A$
3. $cosec^2 A = 1 + cot^2 A$
4. $sin(A \pm B) = sin\,A\,cos\,B \pm cos\,A\,sin\,B$
5. $cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$
6. $tan(A \pm B) = \dfrac{tan\,A \pm tan\,B}{1 \mp tan\,A\,tan\,B}$
7. $sin\,2A = 2\,sin\,A\,cos\,A$
8. $cos\,2A = cos^2 A - sin^2 A$
   $= 2\,cos^2 A - 1$
   $= 1 - 2\,sin^2 A$
9. $tan\,2A = \dfrac{2\,tan\,A}{1 - tan^2\,A}$

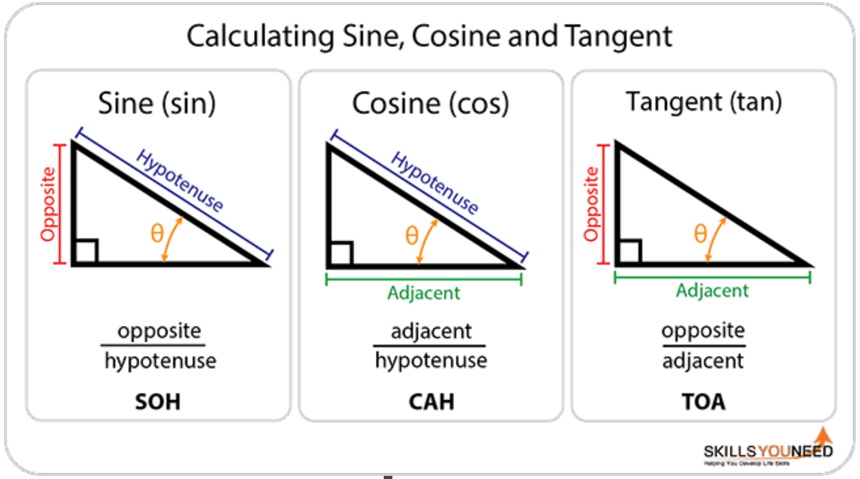
Half-angle formulae:-

10. $sin\,\dfrac{A}{2} = \pm\sqrt{\dfrac{1 - cos\,A}{2}}$
11. $cos\,\dfrac{A}{2} = \pm\sqrt{\dfrac{1 + cos\,A}{2}}$
12. $tan\,\dfrac{A}{2} = \dfrac{sin\,A}{1 + cos\,A}$

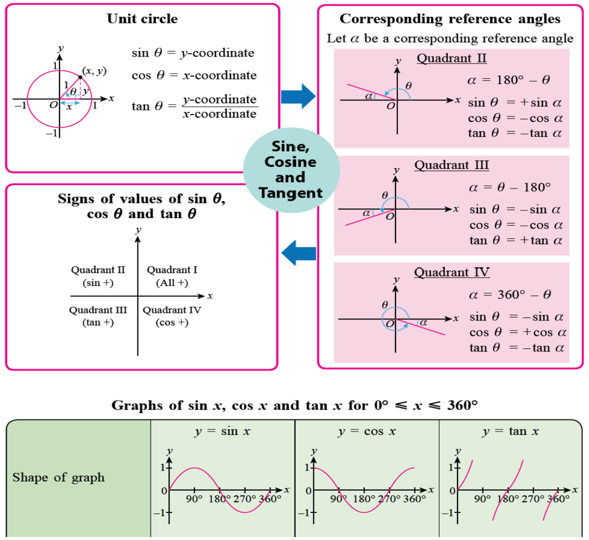
1



2



3



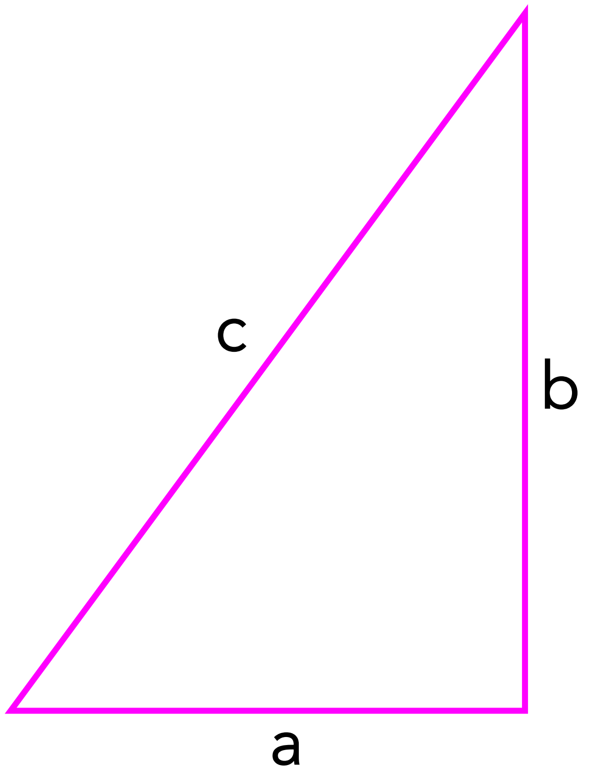
INTRODUCTION

DEFINITION

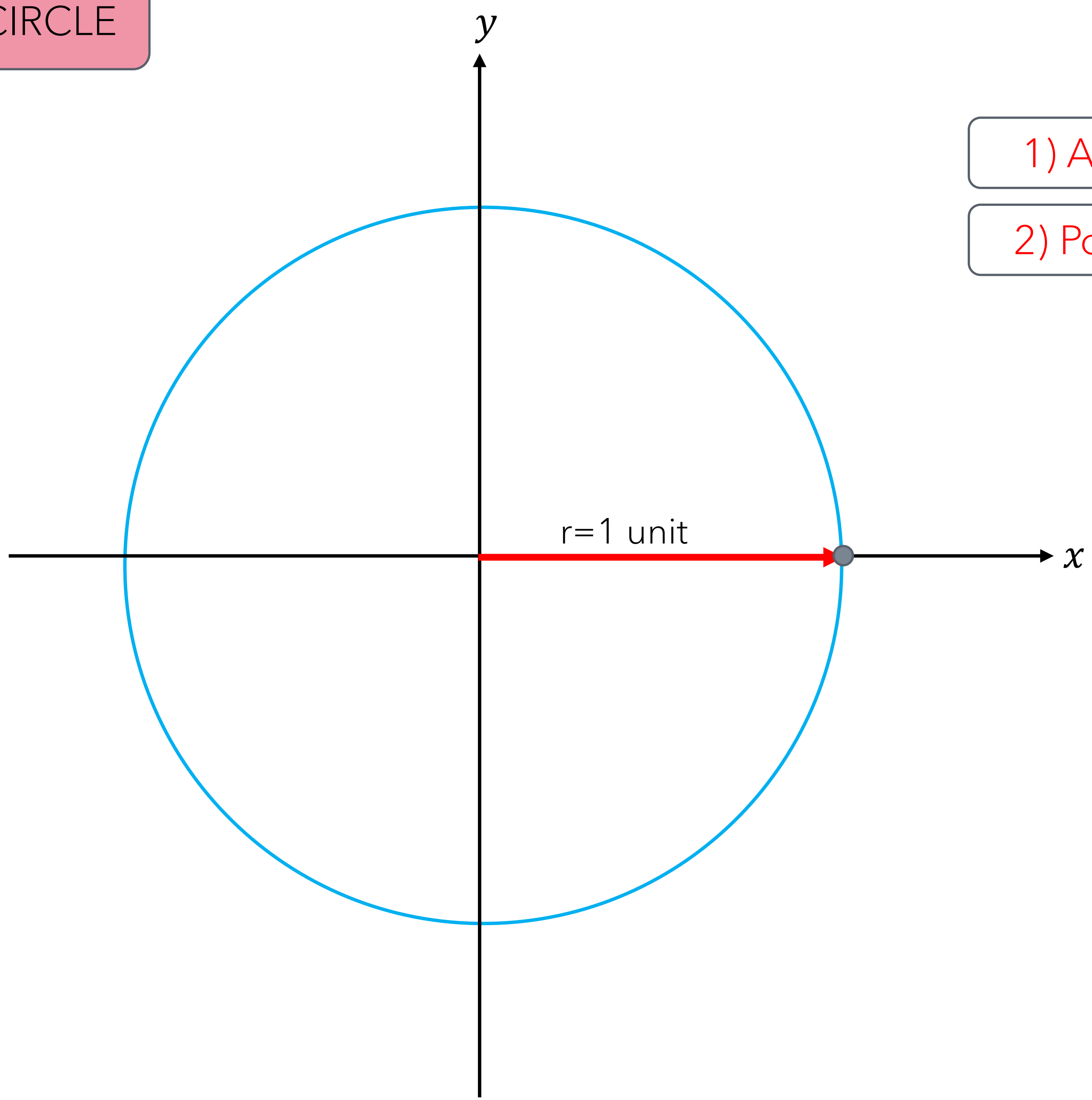
TRIGONOMETRIC

- ‘TRI’ — three
- ‘GONON’ — angle
- ‘METRON’ — measurement

input → FUNCTION → output



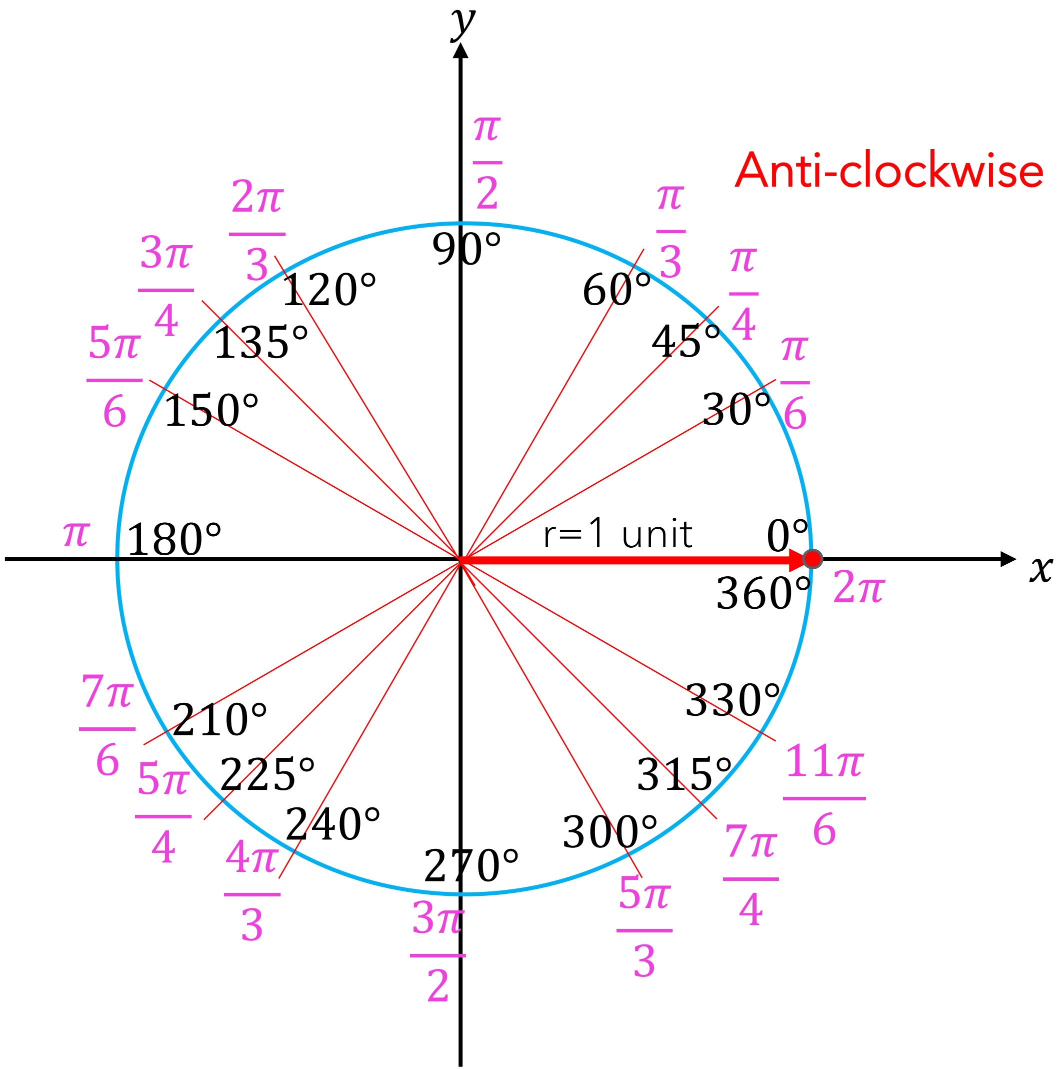
CONSTRUCTION OF A UNIT CIRCLE



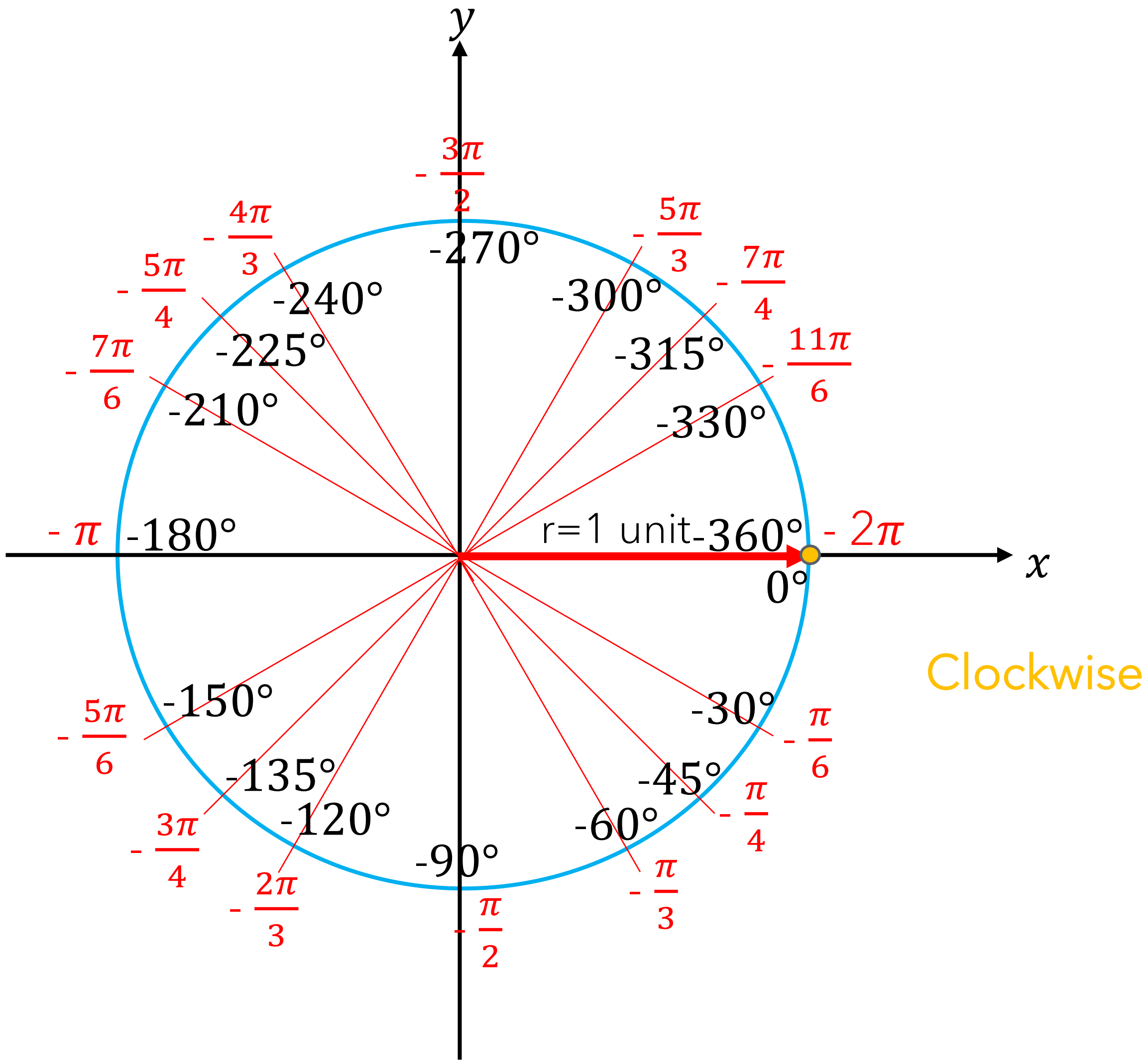
1) Angles

2) Position

POSITIVE AND NEGATIVE ANGLES



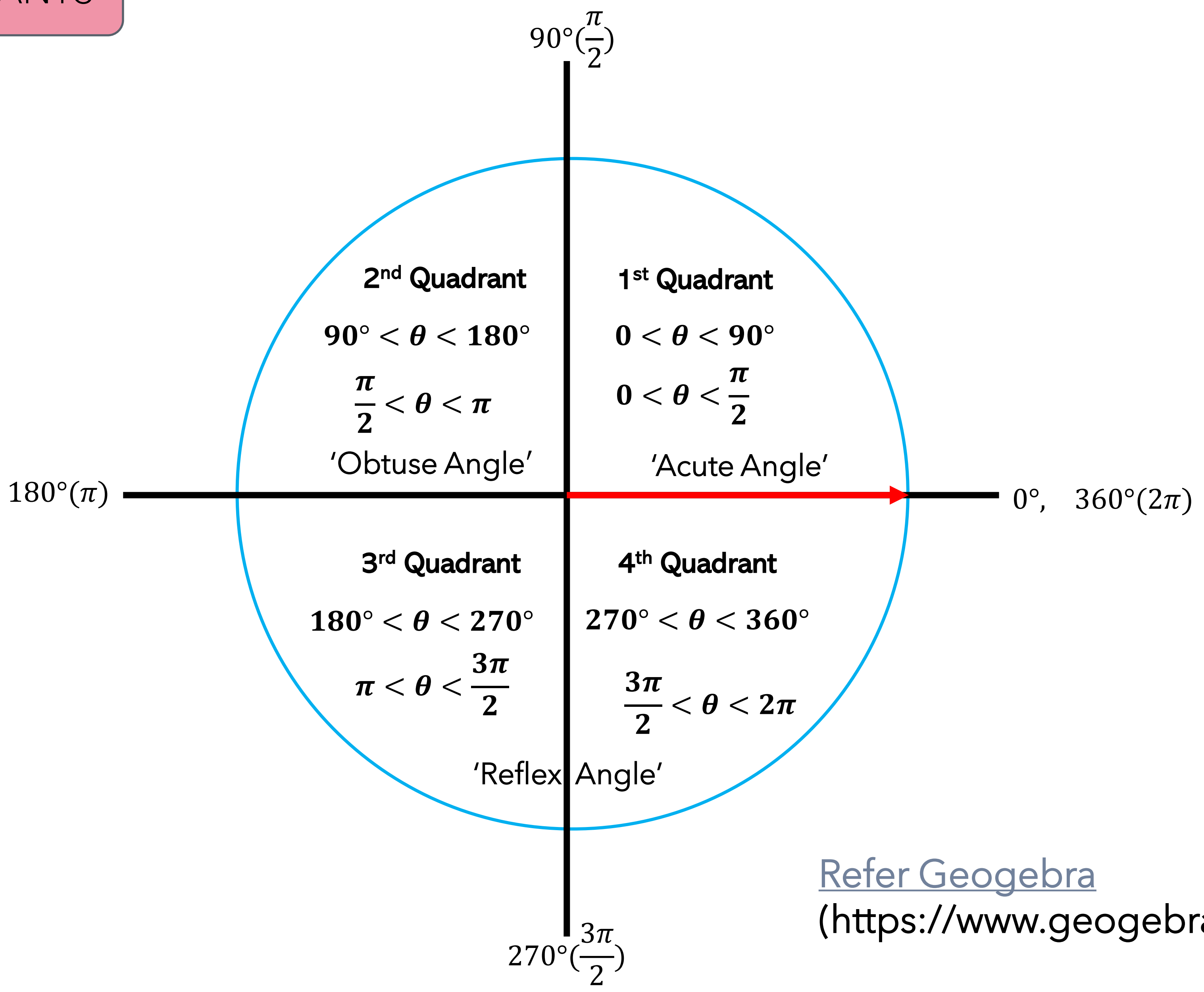
POSITIVE ANGLES



NEGATIVE ANGLES

UNIT CIRCLES

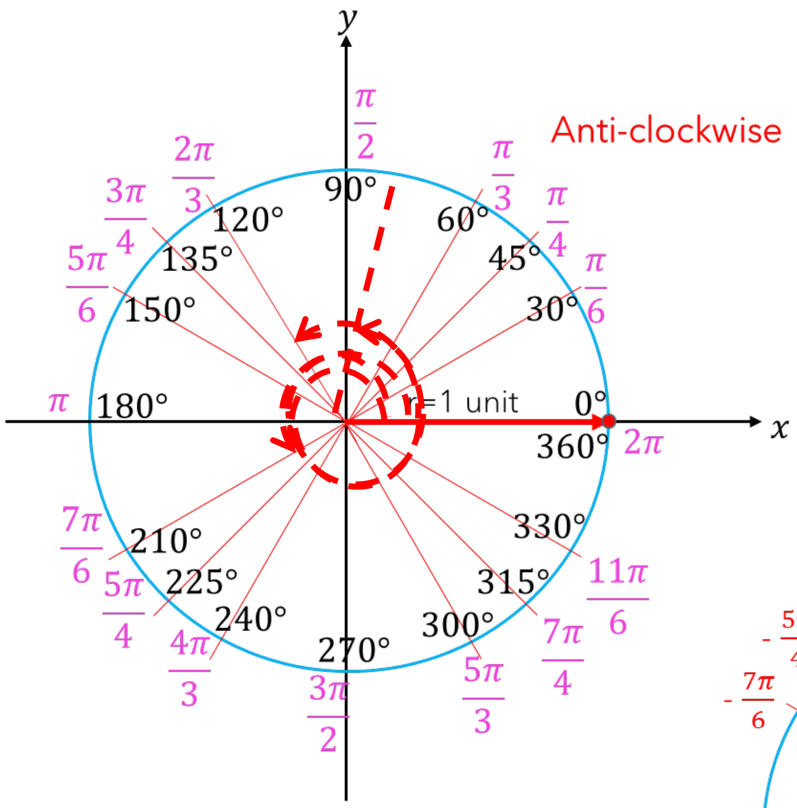
ANGLES AND QUADRANTS



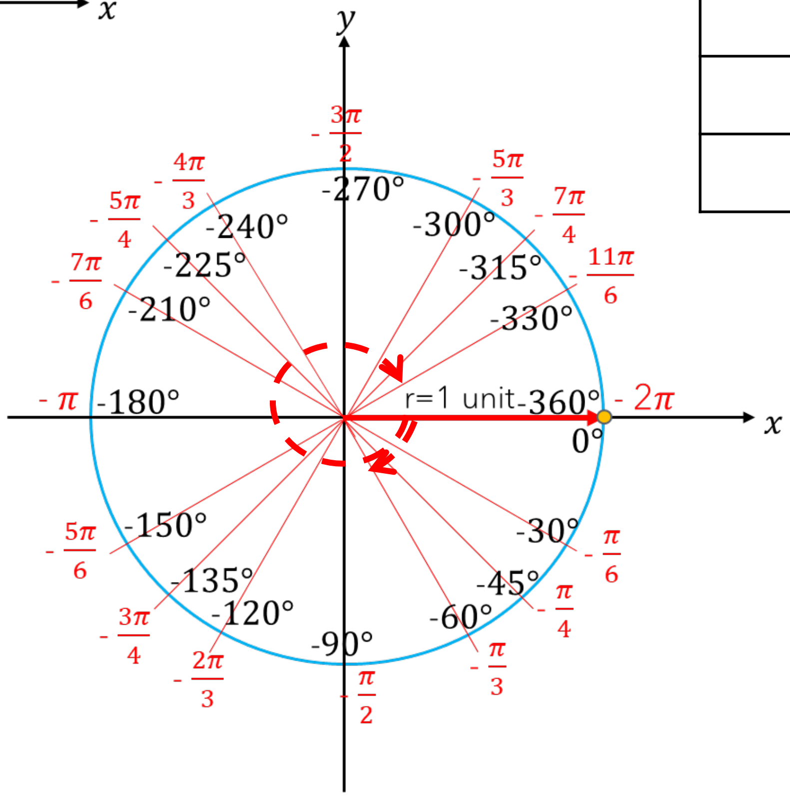
Refer Geogebra
(https://www.geogebra.org/m/zubdsuy4)

Example 1: State the quadrant for each angle below.

Angle	Quadrant
$210°$	3
$-330°$	1
$-\frac{1}{3}\pi$ rad	4
$440°$	1
$840°$	2



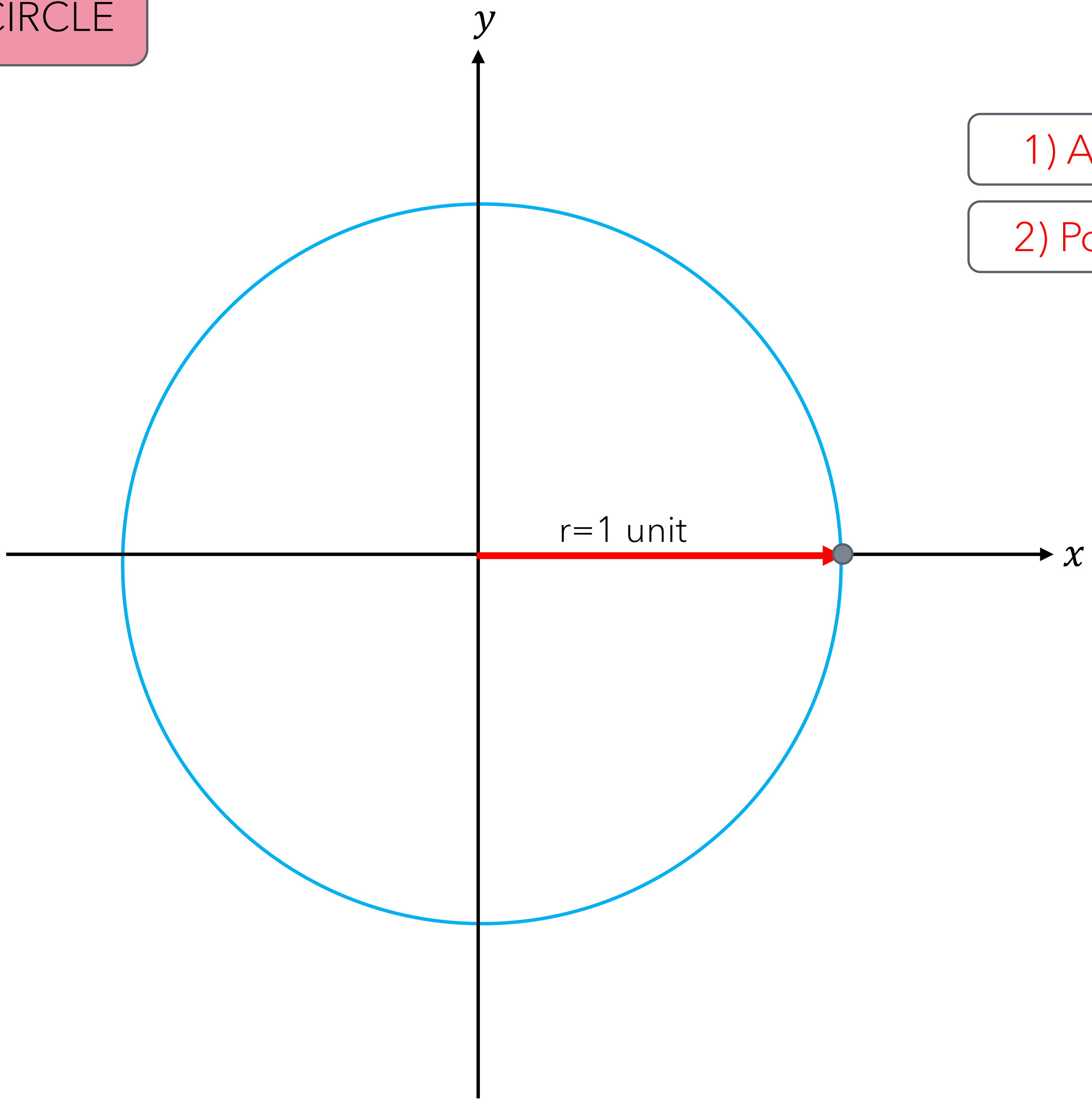
POSITIVE ANGLES



NEGATIVE ANGLES

UNIT CIRCLES

CONSTRUCTION OF A UNIT CIRCLE

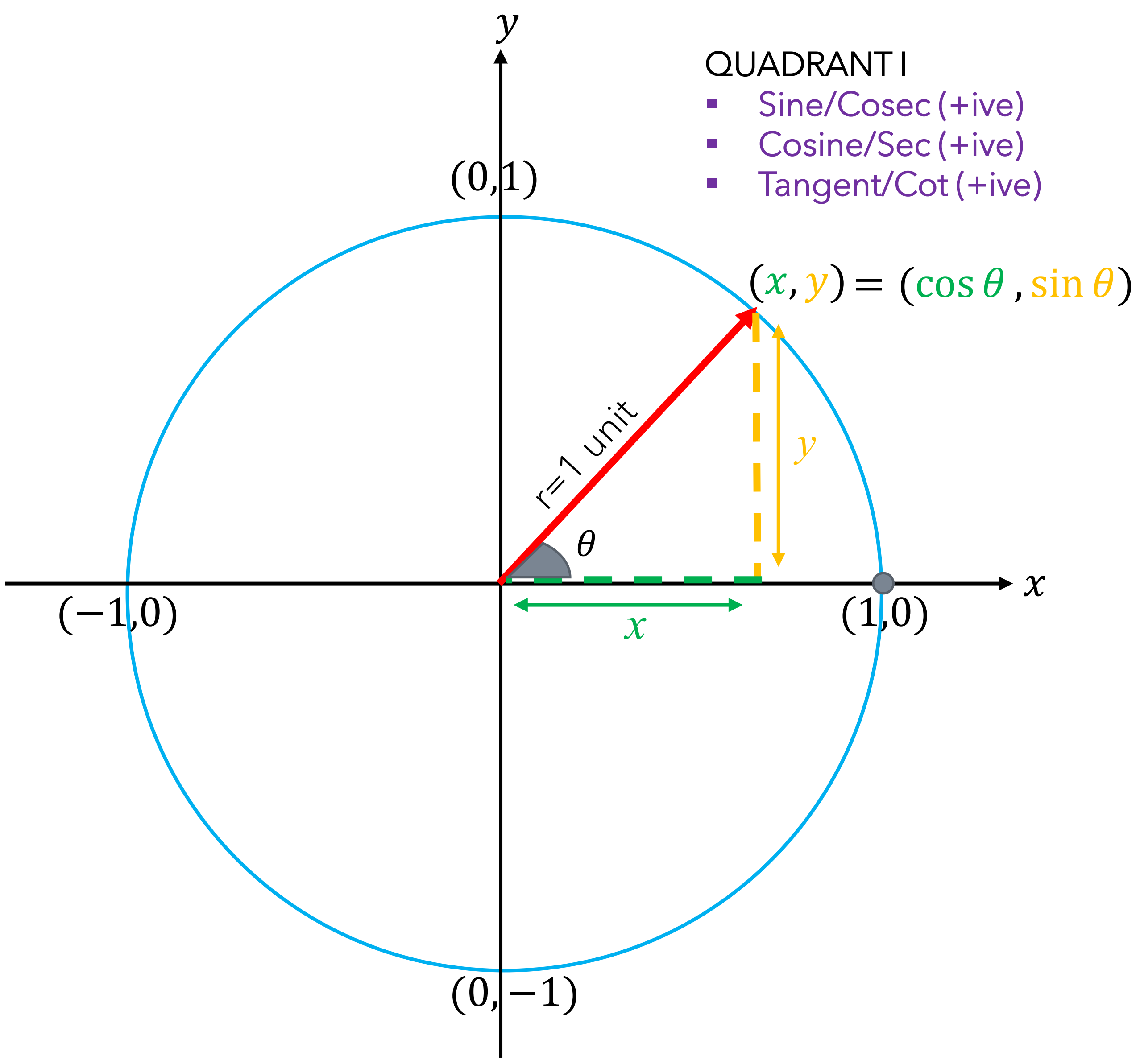


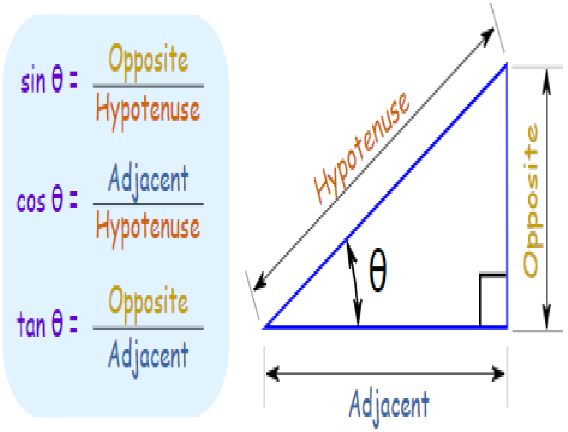
1) Angles ✓

2) Position ✓

1ST QUADRANT

QUADRANT I
- Sine/Cosec (+ive)
- Cosine/Sec (+ive)
- Tangent/Cot (+ive)





Reciprocal Trig Ratios
$cosecant = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$
$secant = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent}$
$cotangent = \frac{1}{tan} = \frac{adjacent}{opposite}$

$$\sin\theta = \frac{y}{1} \quad \therefore y = \sin\theta$$

$$\cos\theta = \frac{x}{1} \quad \therefore x = \cos\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$$

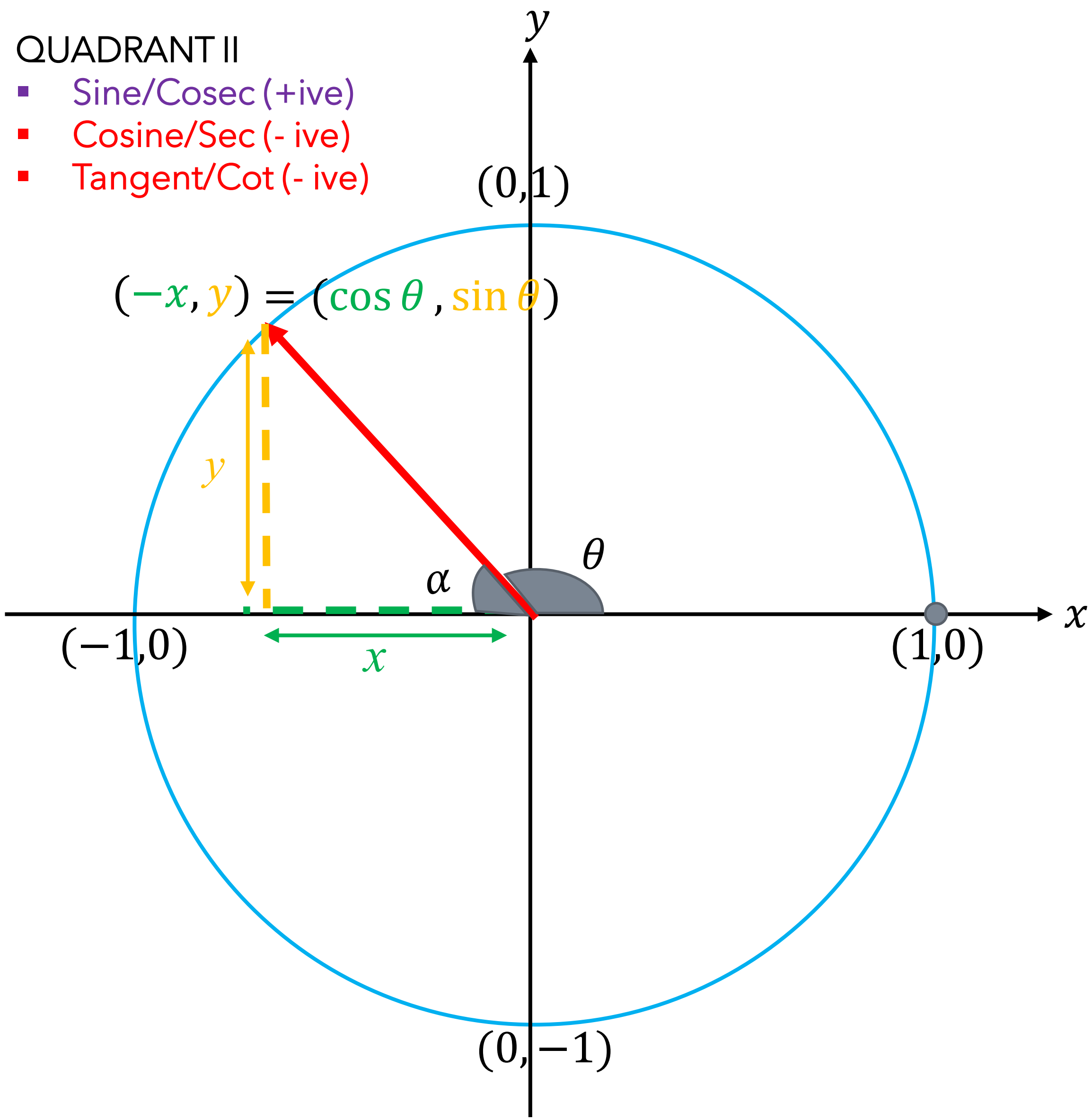
$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$$

TRIGON. RATIO

2ND QUADRANT

QUADRANT II

- Sine/Cosec (+ive)
- Cosine/Sec (- ive)
- Tangent/Cot (- ive)



$\sin\alpha = \frac{y}{1}$ $\therefore \sin\theta = y$

$\cos\alpha = \frac{x}{1}$ $\therefore \cos\theta = -x$

$\tan\alpha = \frac{y}{x}$ $\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{y}{x}$

$\text{cosec}\,\alpha = \frac{1}{y}$ $\therefore \text{cosec}\,\theta = \frac{1}{y}$

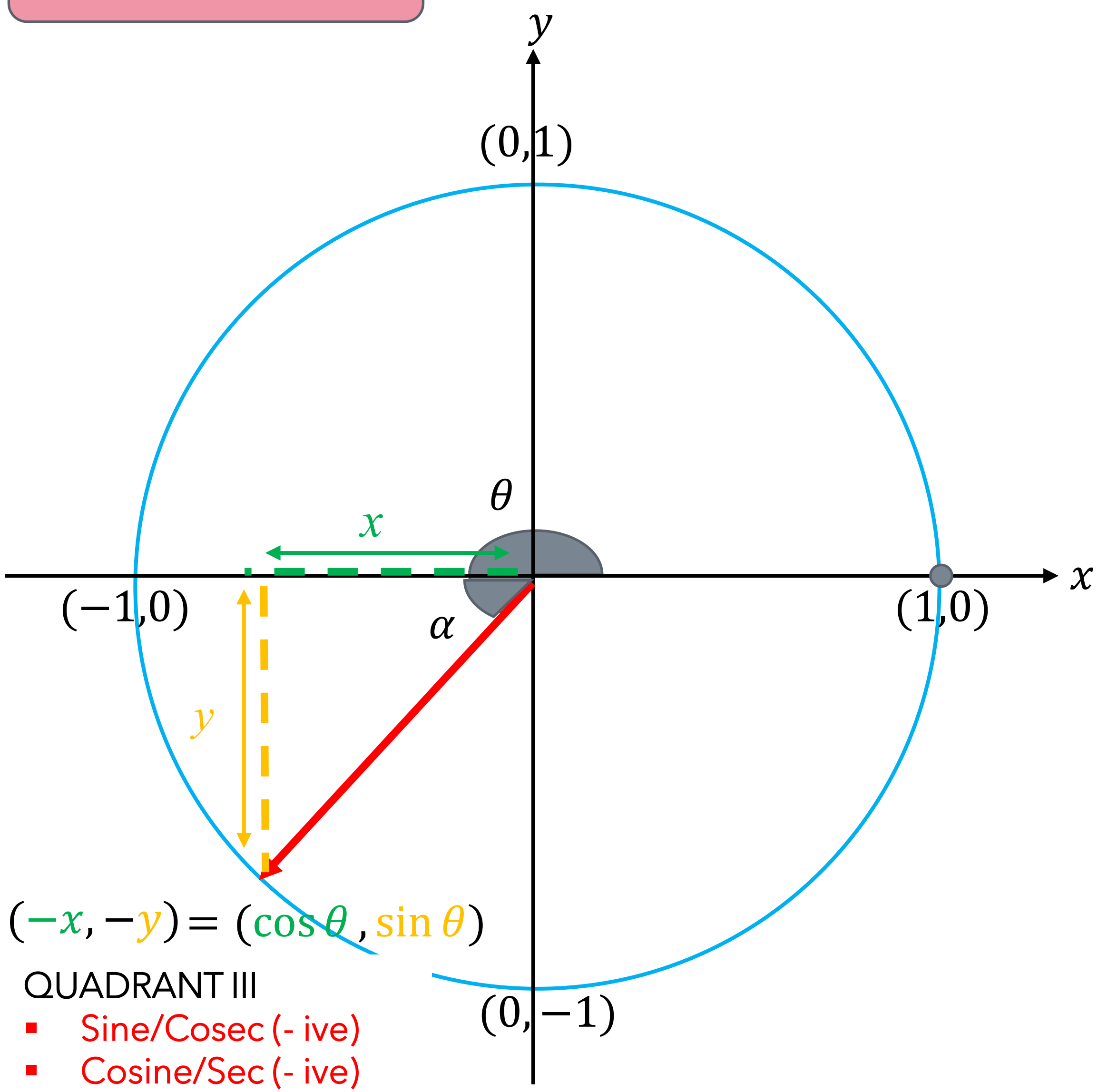
$\sec\alpha = \frac{1}{x}$ $\therefore \sec\theta = -\frac{1}{x}$

$\cot\alpha = \frac{x}{y}$ $\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = -\frac{x}{y}$

Refer Geogebra
(https://www.geogebra.org/m/zubdsuy4)

TRIGON. RATIO

3RD QUADRANT



$(-x, -y) = (\cos\theta, \sin\theta)$

QUADRANT III

- Sine/Cosec (- ive)
- Cosine/Sec (- ive)
- Tangent/Cot (+ ive)

$\sin\alpha = \frac{y}{1}$ $\quad \therefore \sin\theta = -y$

$\cos\alpha = \frac{x}{1}$ $\quad \therefore \cos\theta = -x$

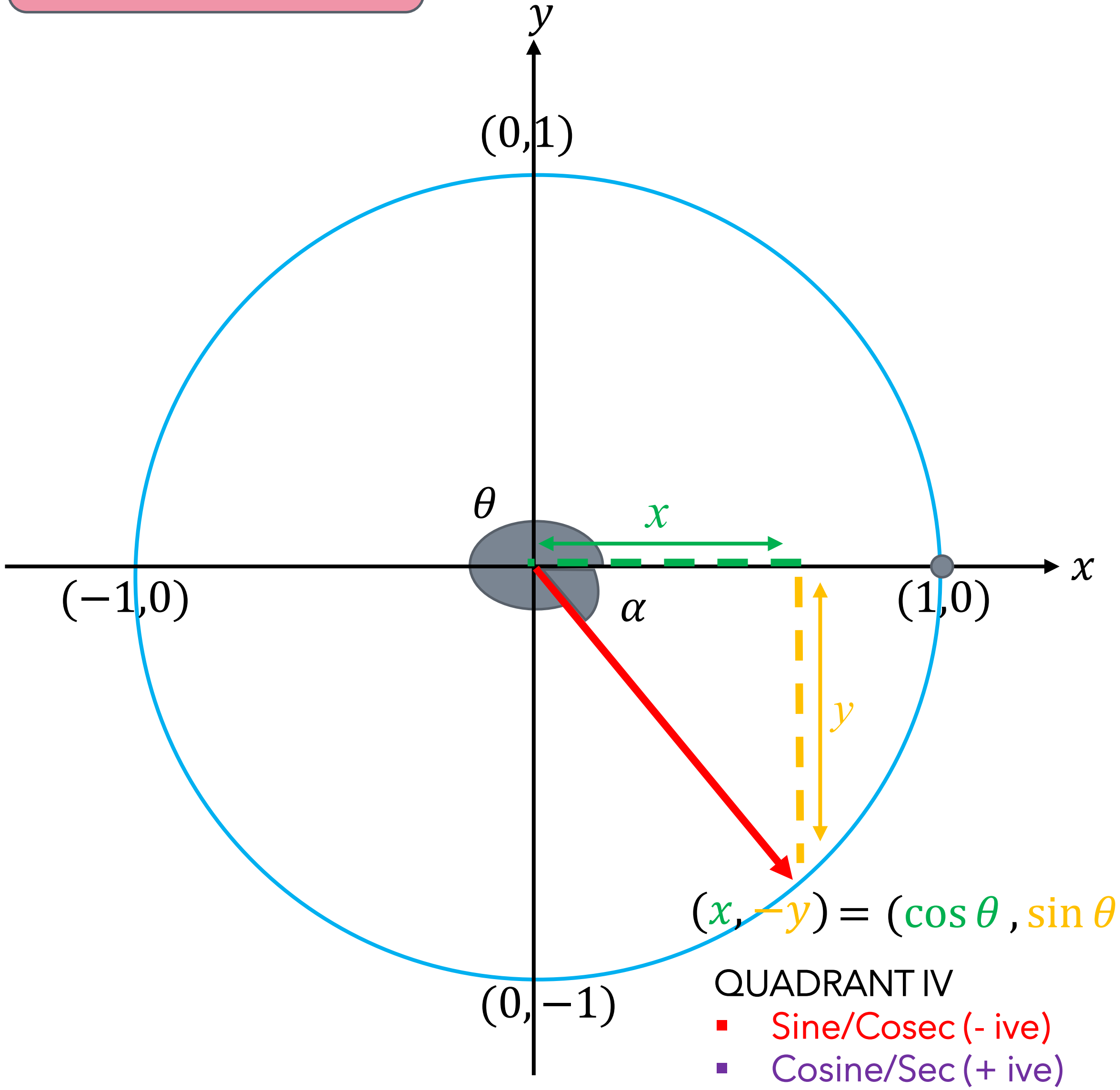
$\tan\alpha = \frac{y}{x}$ $\quad \therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

$\mathrm{cosec}\,\alpha = \frac{1}{y}$ $\quad \therefore \mathrm{cosec}\,\theta = -\frac{1}{y}$

$\sec\alpha = \frac{1}{x}$ $\quad \therefore \sec\theta = -\frac{1}{x}$

$\cot\alpha = \frac{x}{y}$ $\quad \therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$

4TH QUADRANT



$(x, -y) = (\cos\theta, \sin\theta)$

QUADRANT IV
- Sine/Cosec (- ive)
- Cosine/Sec (+ ive)
- Tangent/Cot (- ive)

$\sin\alpha = \frac{y}{1}$ $\quad \therefore \sin\theta = -y$

$\cos\alpha = \frac{x}{1}$ $\quad \therefore \cos\theta = x$

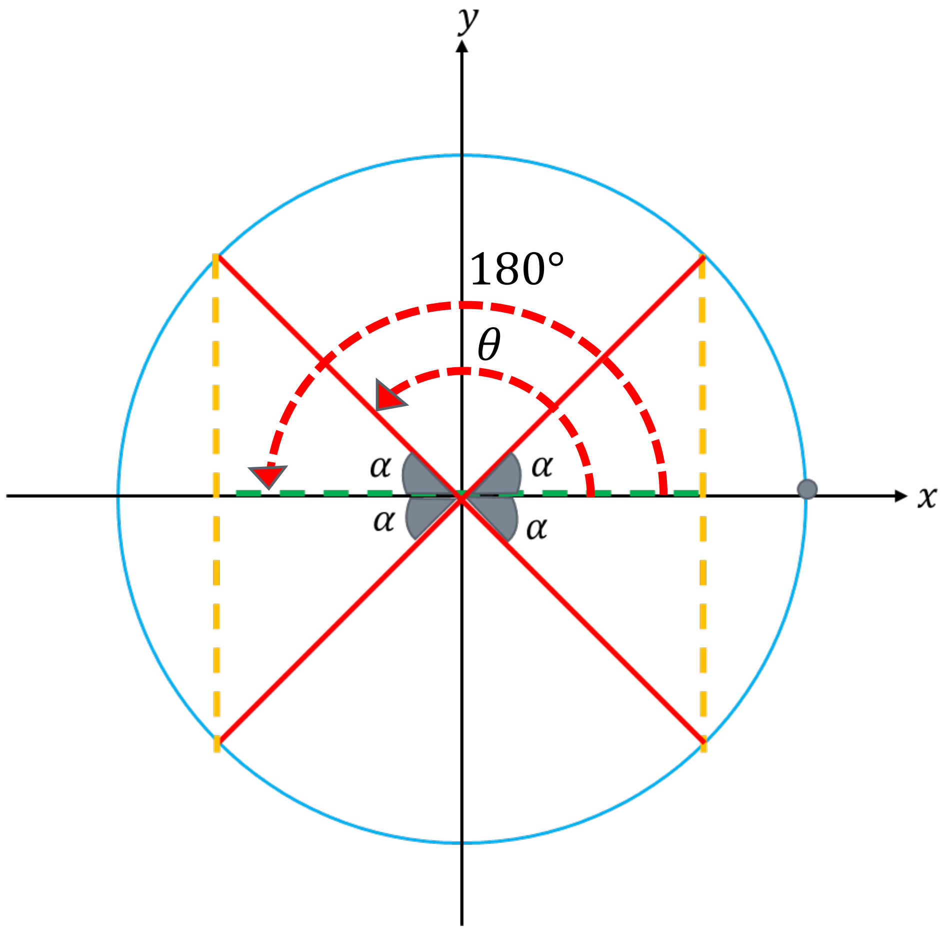
$\tan\alpha = \frac{y}{x}$ $\quad \therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{y}{x}$

$\text{cosec}\,\alpha = \frac{1}{y}$ $\quad \therefore \text{cosec}\,\theta = -\frac{1}{y}$

$\sec\alpha = \frac{1}{x}$ $\quad \therefore \sec\theta = \frac{1}{x}$

$\cot\alpha = \frac{x}{y}$ $\quad \therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = -\frac{x}{y}$

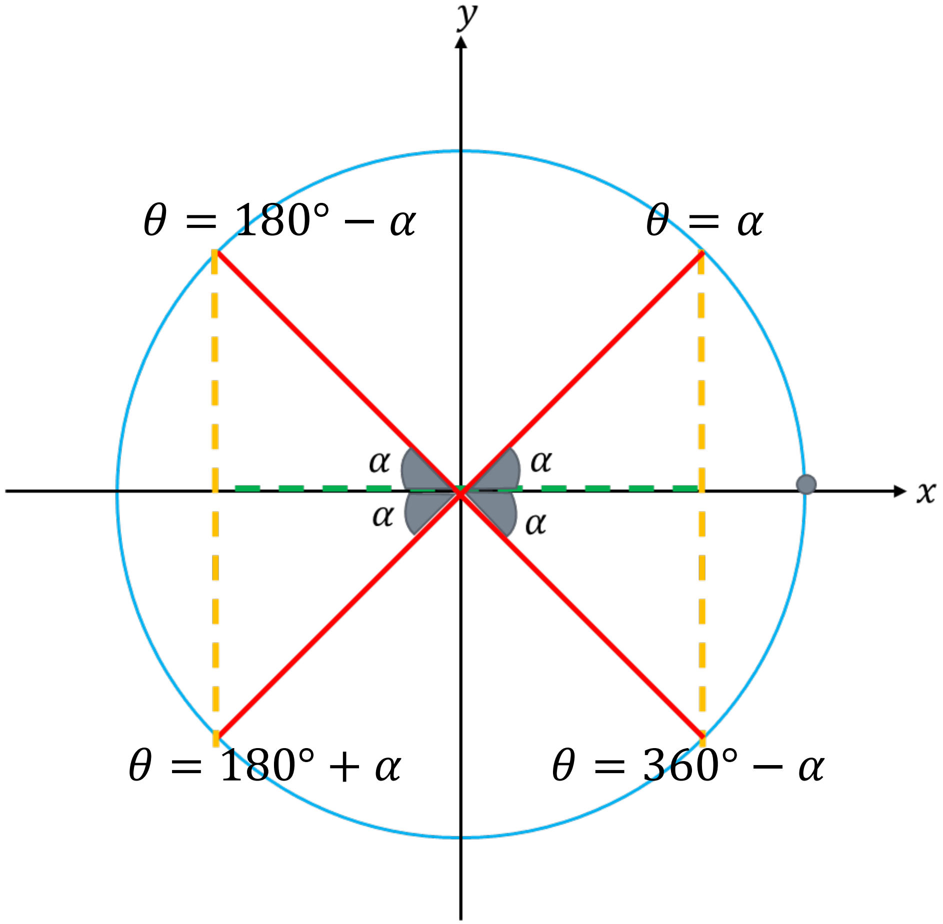
REFERENCE ANGLE



Example 2: For each of the following angle, state the quadrant and reference angle.

Angle, θ	Quadrant	Reference Angle, α
$100°$	2	$80°$
$200°$	3	$20°$
$400°$	1	$40°$
$300°$	4	$60°$

CORRESPONDING ANGLE

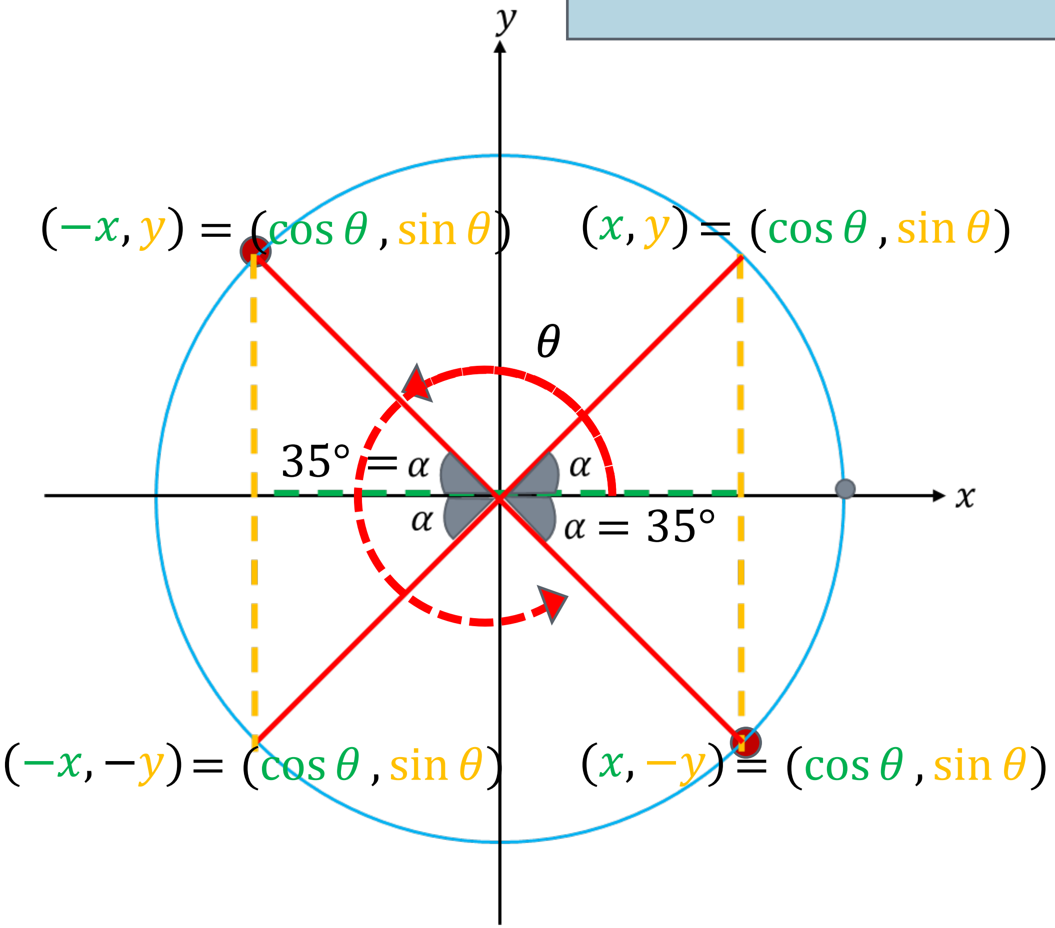


Example 3: Given the reference angles and its quadrant. State their corresponding angle.

Quadrant	Reference Angle, α	Corresponding Angle, θ
I	55°	55°
II	30°	150°
III	45°	225°
IV	75°	285°

1) Quadrant
2) Reference Angle
3) Corresponding Angle

1) Quadrant
2) Reference Angle
3) Corresponding Angle



Example 4: Solve $tan\,\theta = -0.7$ for $0° \leq \theta \leq 360°$

$tan\,\theta = -0.7$

Reference angle, α

$\alpha = tan^{-1}0.7$

$\alpha = 35°$

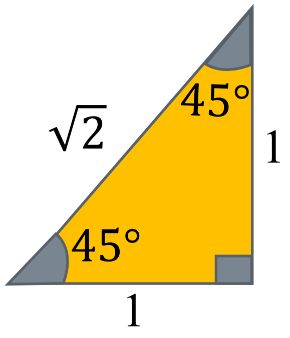
Corresponding angles, θ

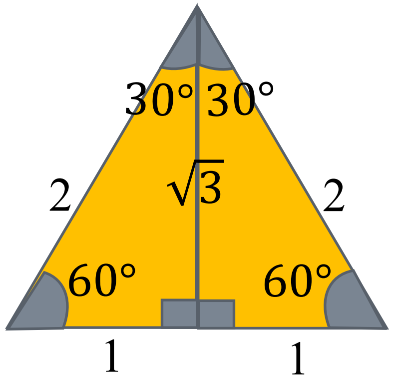
$\theta = (180° - 35°), (360° - 35°)$

$\theta = 145°, 325°$

SPECIAL ANGLES (0°, 30°, 45°, 60°, 90°)

Example 5: Find the value of trigonometric ratio for each special angle below. Express your answer in its simplest form.





a) $\sin 60°$

$$= \frac{\sqrt{3}}{2}$$

b) $\cos 45°$

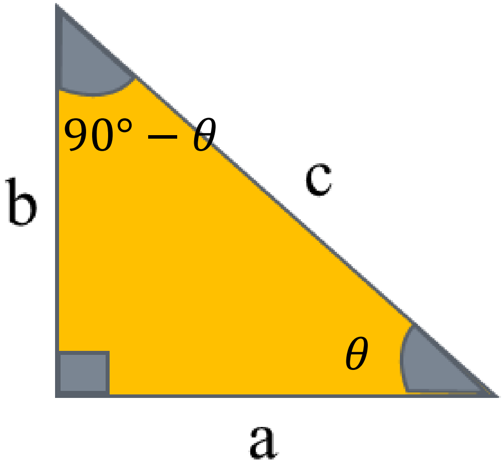
$$= \frac{1}{\sqrt{2}}$$

c) $\tan \frac{\pi}{6}$

$$= \frac{1}{\sqrt{3}}$$

Angle \ Ratio		sin	cos	tan	cosec	sec	cot
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$

COMPLEMENTARY ANGLES



$\sin\theta = \frac{b}{c}$	$\sin(90° - \theta) = \frac{a}{c}$
$\cos\theta = \frac{a}{c}$	$\cos(90° - \theta) = \frac{b}{c}$
$\sec\theta = \frac{c}{a}$	$\sec(90° - \theta) = \frac{c}{b}$
$\text{cosec}\,\theta = \frac{c}{b}$	$\text{cosec}(90° - \theta) = \frac{c}{a}$
$\tan\theta = \frac{b}{a}$	$\tan(90° - \theta) = \frac{a}{b}$
$\cot\theta = \frac{a}{b}$	$\cot(90° - \theta) = \frac{b}{a}$

- $\sin\theta = \cos(90° - \theta)$
- $\cos\theta = \sin(90° - \theta)$
- $\tan\theta = \cot(90° - \theta)$
- $\sec\theta = \text{cosec}(90° - \theta)$
- $\text{cosec}\,\theta = \sec(90° - \theta)$
- $\cot\theta = \tan(90° - \theta)$

COMPLEMENTARY ANGLES

Assume 'co' stands for complementary

co s – 'complementary' of sine
co sec – 'complementary' of secant
co t – 'complementary' of tangent

Example 6: Given $\cos 25° = 0.9063$, and $\sin 25° = 0.4226$. Without using calculator, find the value of each of the following

a) $\sin 65°$

$= \cos(90° - 65°)$

$= \cos 25°$

$= 0.9063$

b) $\sec 65°$

$= cosec(90° - 65°)$

$= \text{cosec}\, 25°$

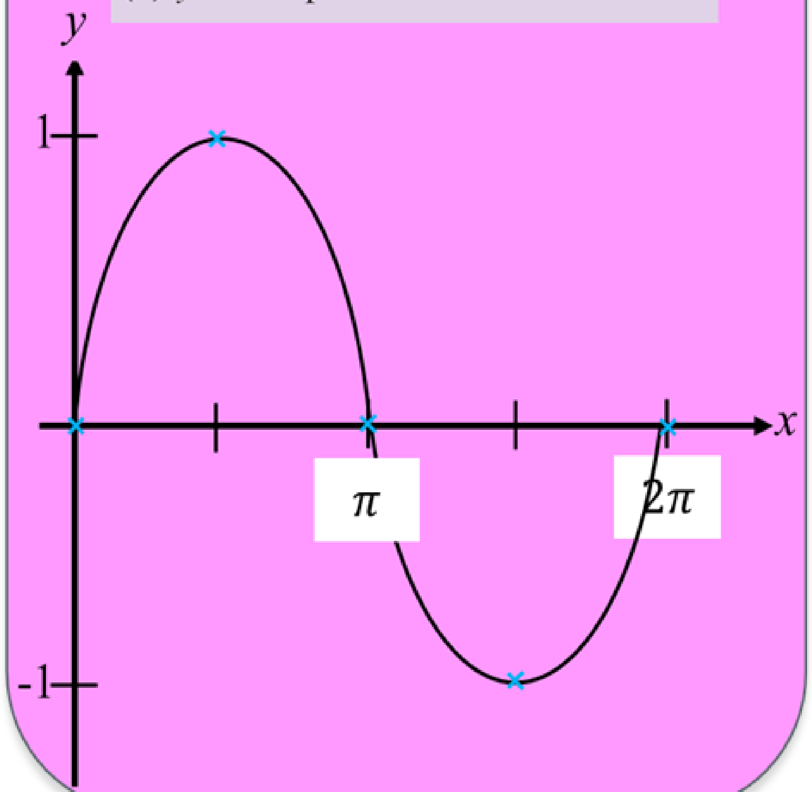
$$= \frac{1}{sin25°}$$

$$= \frac{1}{0.4226}$$

BASIC GRAPHS

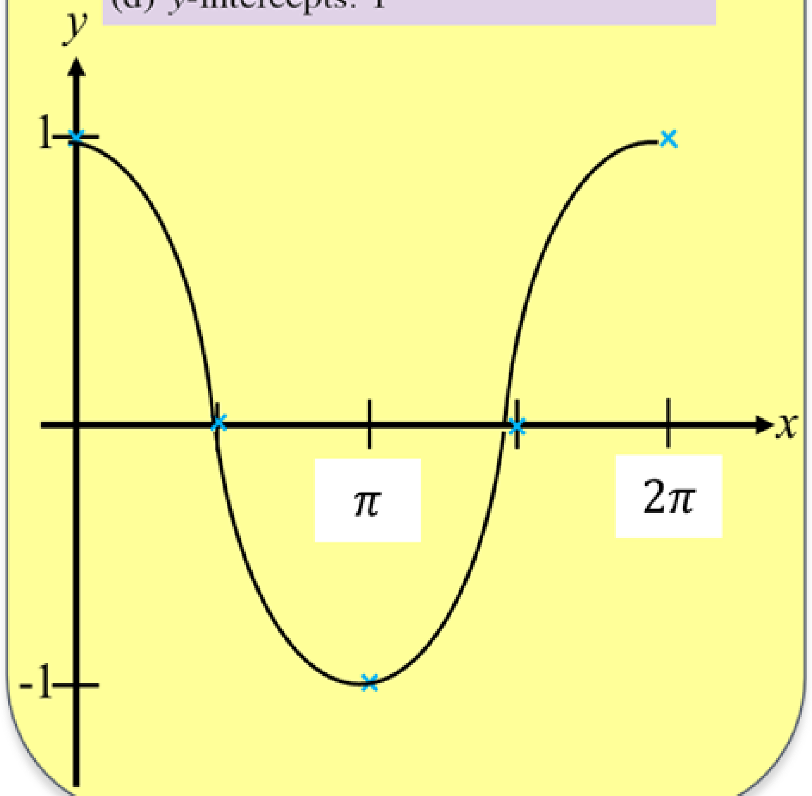
Example 7: $y = \sin x$

Graph $y = \sin x$ for $-2\pi \leqslant x \leqslant 2\pi$
(a) Amplitude = 1
  (i) The maximum value of $y = 1$
  (ii) The minimum value of $y = -1$
(b) Period = 360° or 2π
(c) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$
(d) y-intercepts: 0



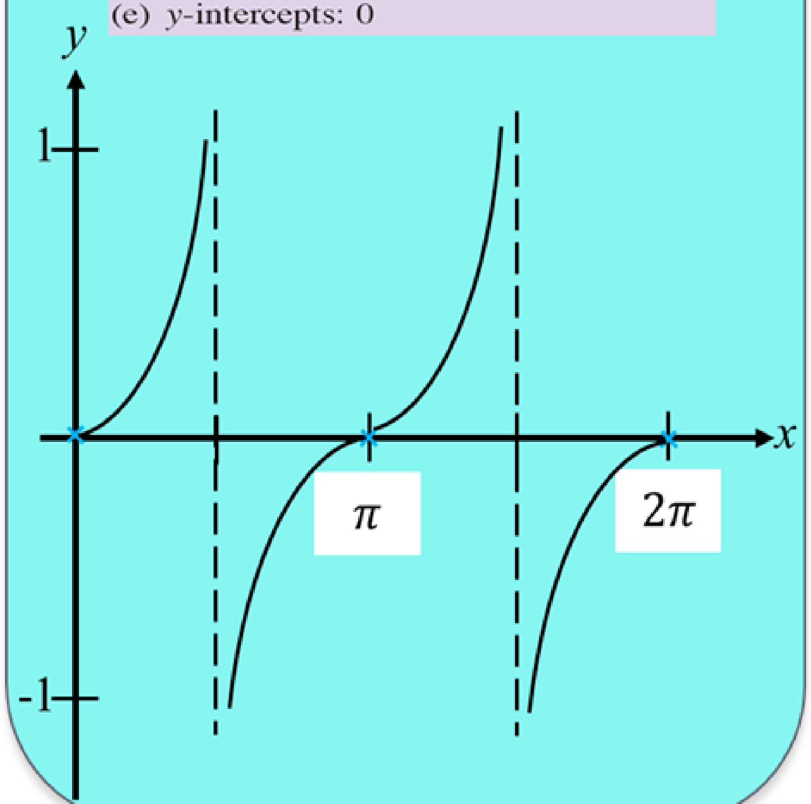
Example 8: $y = \cos x$

Graph $y = \cos x$ for $-2\pi \leqslant x \leqslant 2\pi$
(a) Amplitude = 1
  (i) The maximum value of $y = 1$
  (ii) The minimum value of $y = -1$
(b) Period = 360° or 2π
(c) x-intercepts: $-\frac{3}{2}\pi, -\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{3}{2}\pi$
(d) y-intercepts: 1



Example 9: $y = \tan x$

Graph $y = \tan x$ for $-2\pi \leqslant x \leqslant 2\pi$
(a) No amplitude
  (i) There is no maximum value of y
  (ii) There is no minimum value of y
(b) Period = 180° or π
(c) x-asymptotes: $-\frac{3}{2}\pi, -\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{3}{2}\pi$
(d) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$
(e) y-intercepts: 0

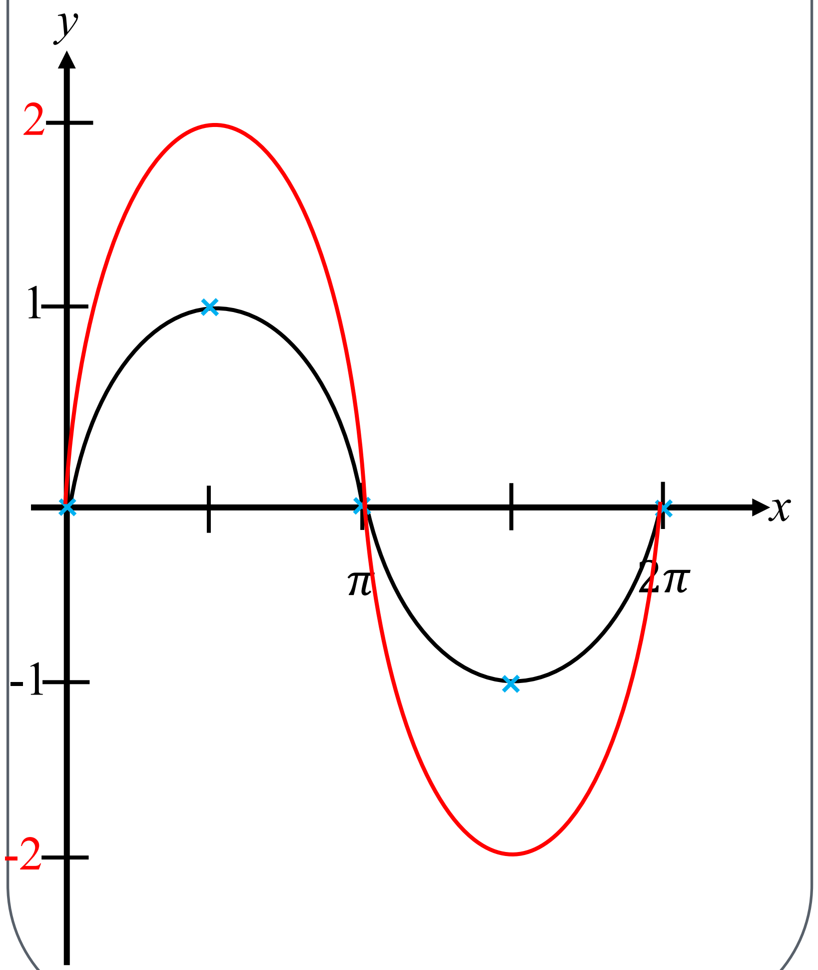


TRIGON. RATIO

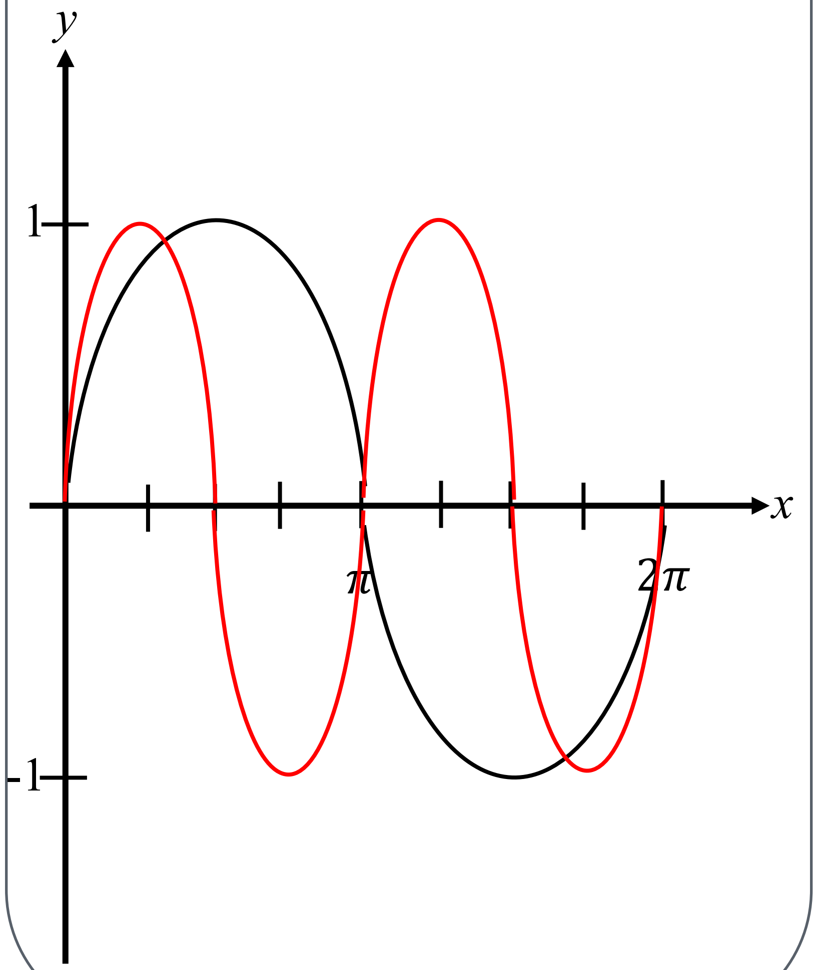
Effect of changing **a**, **b** and **c** for $\boldsymbol{y = a \sin bx + c}$

[Geogebra](https://www.geogebra.org/m/gbnsfqbx) (https://www.geogebra.org/m/gbnsfqbx)

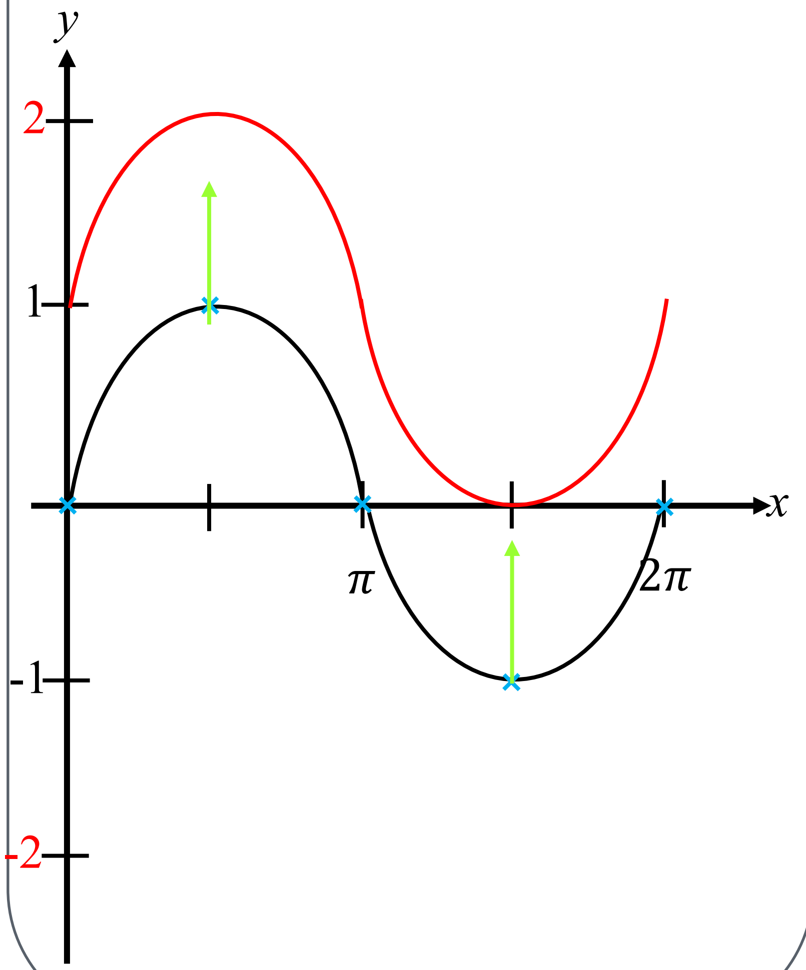
Example 10: $y = 2 \sin x$



Example 11: $y = \sin 2x$



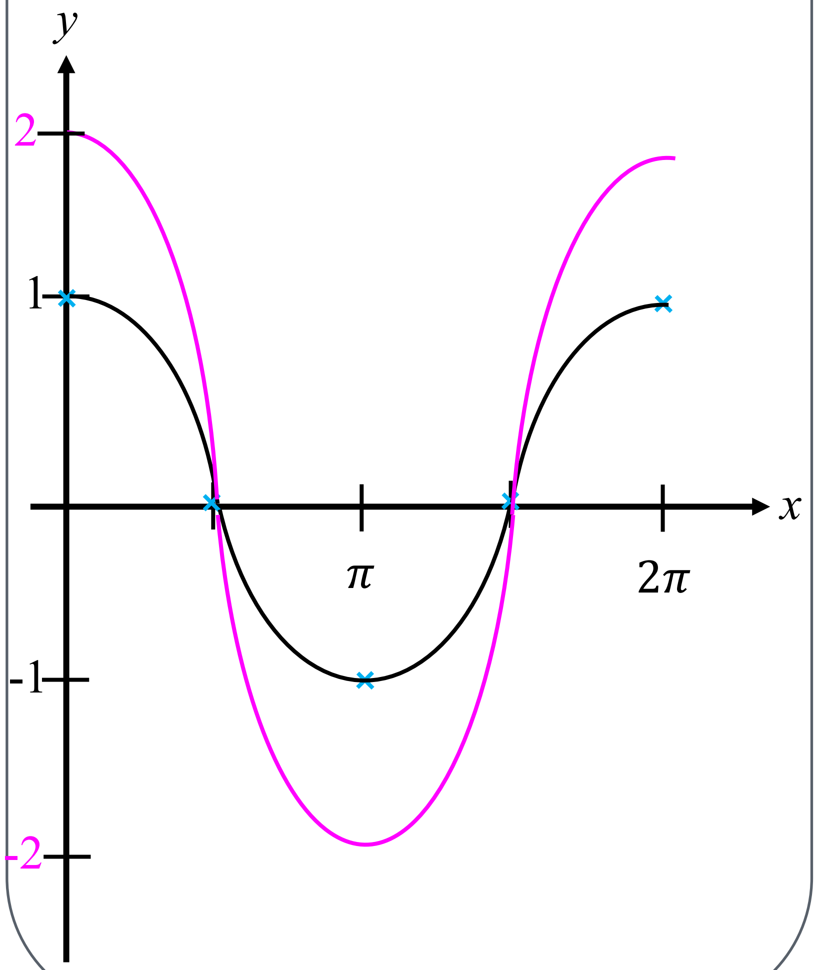
Example 12: $y = \sin x + 1$



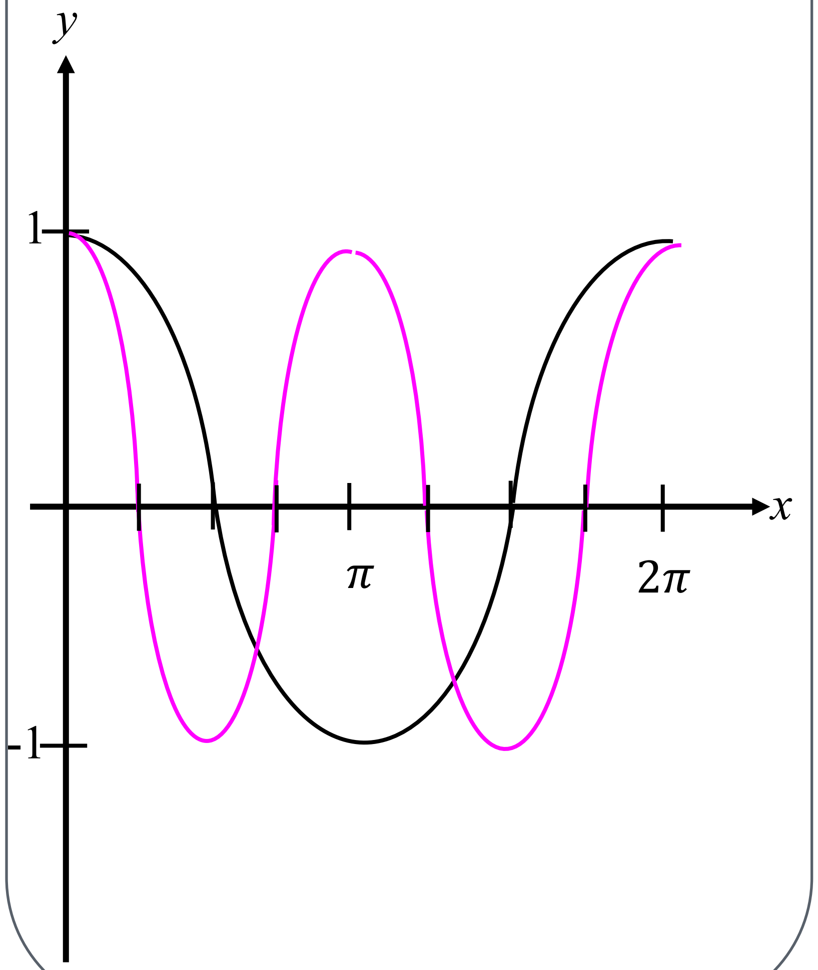
Effect of changing **a**, **b** and **c** for $y = a\cos bx + c$

Geogebra (https://www.geogebra.org/m/knybs53t)

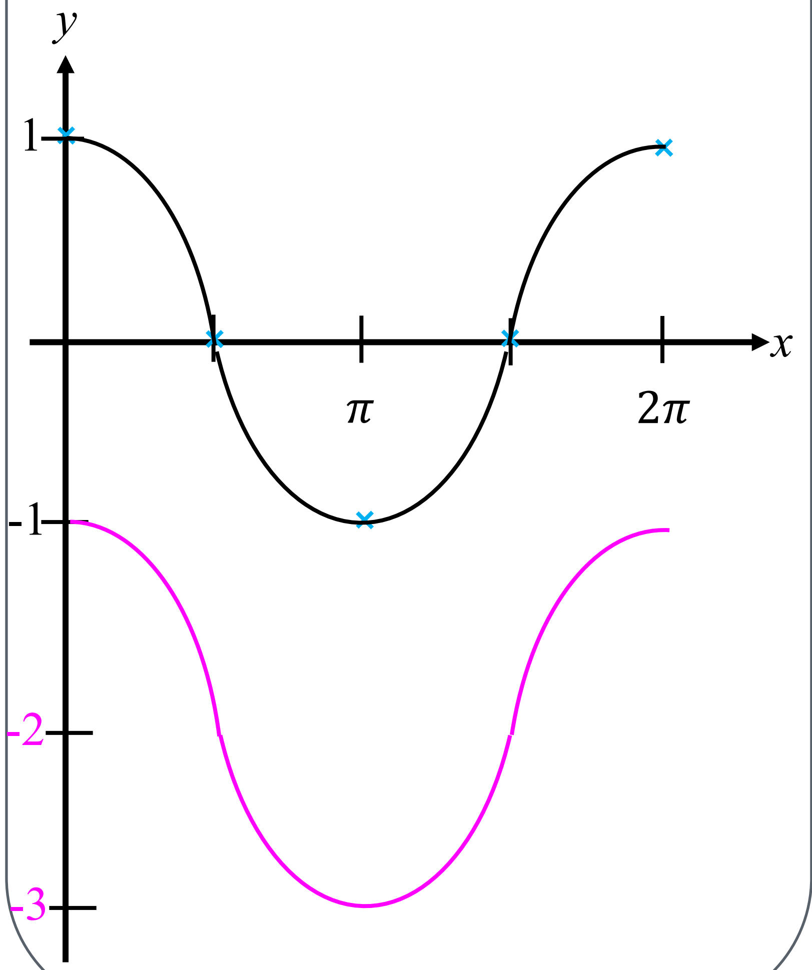
Example 13: $y = 2\cos x$



Example 14: $y = \cos 2x$



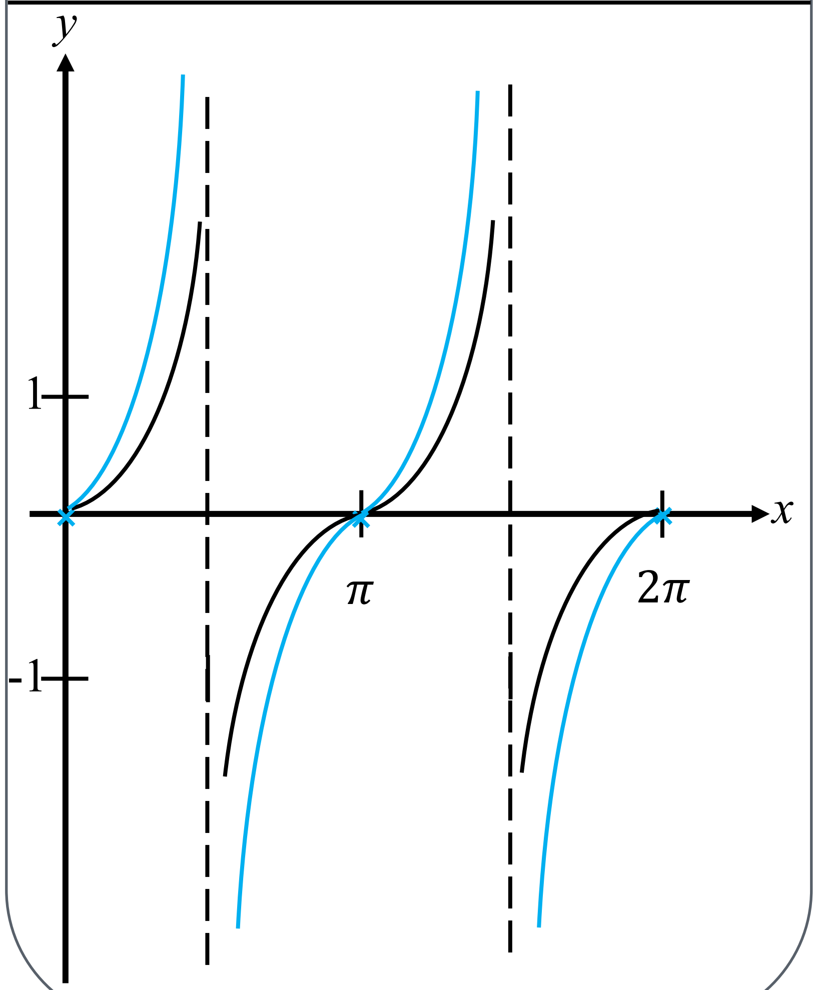
Example 15: $y = \cos x - 2$



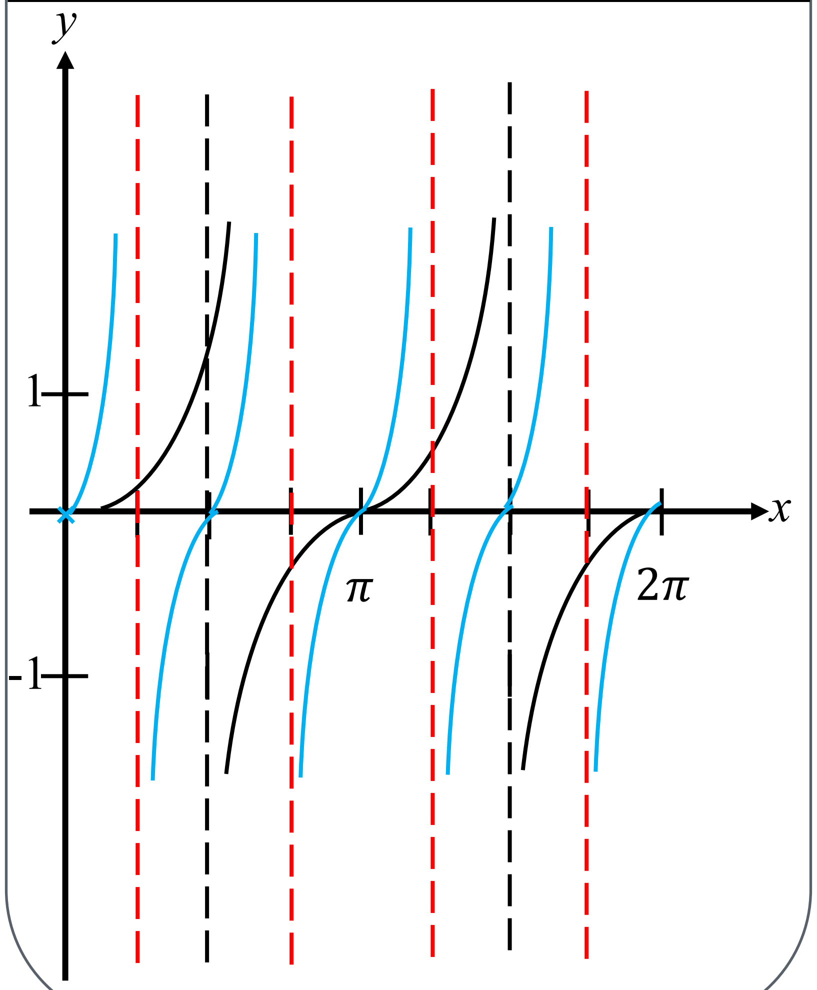
Effect of changing **a**, **b** and **c** for $y = a \tan bx + c$

Geogebra (https://www.geogebra.org/m/vg5hdt3j)

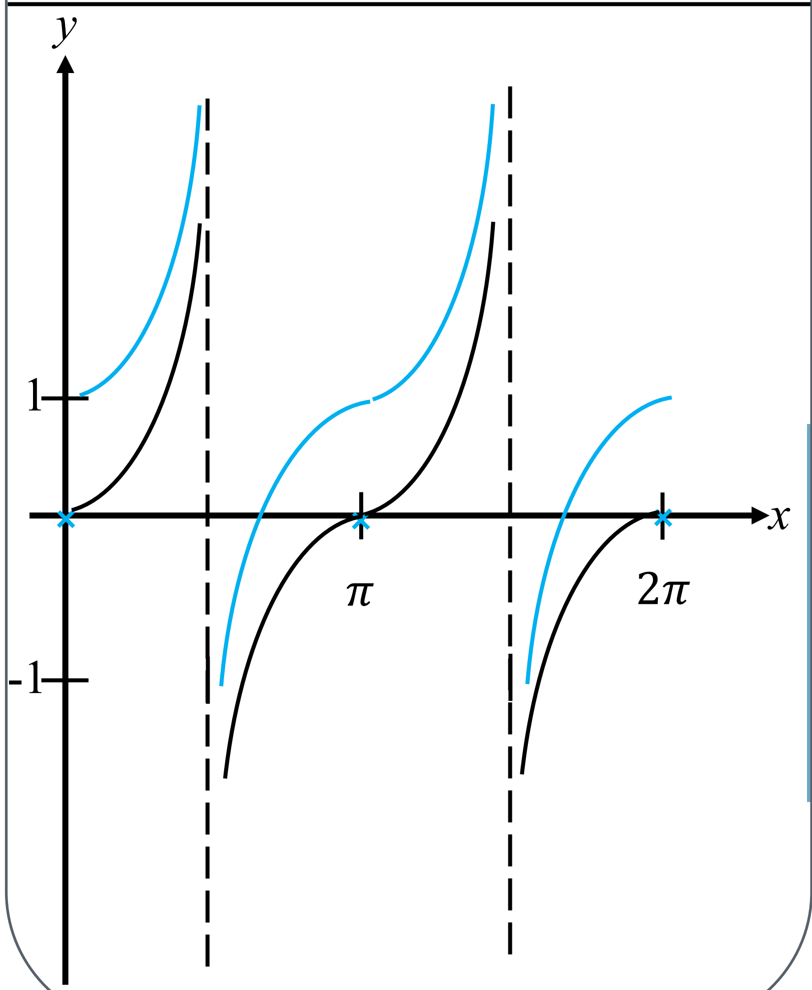
Example 16: $y = 2 \tan x$



Example 17: $y = \tan 2x$



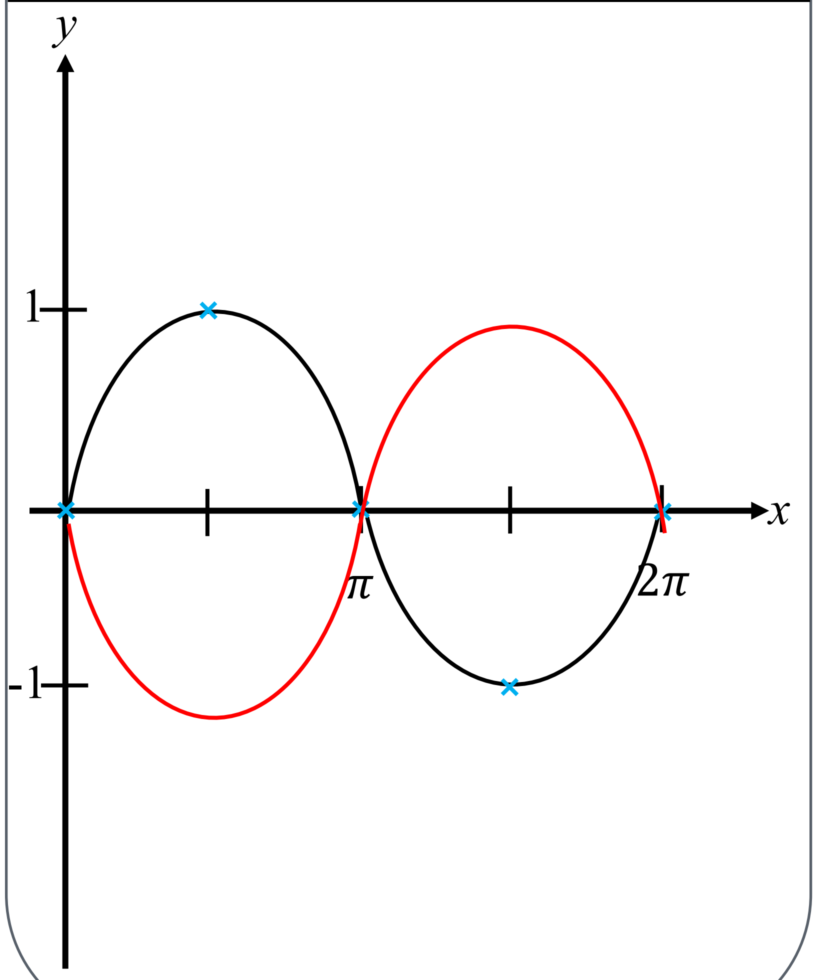
Example 18: $y = \tan x + 1$



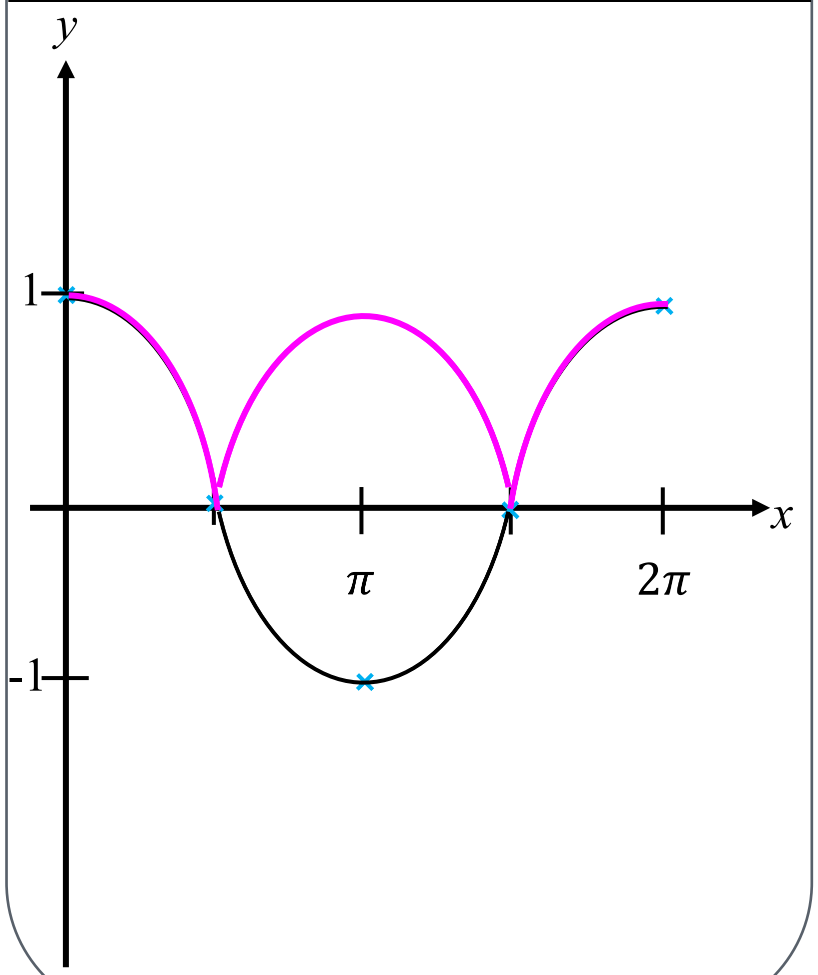
TRIGON. RATIO

Special effects – a & $|modulus|$

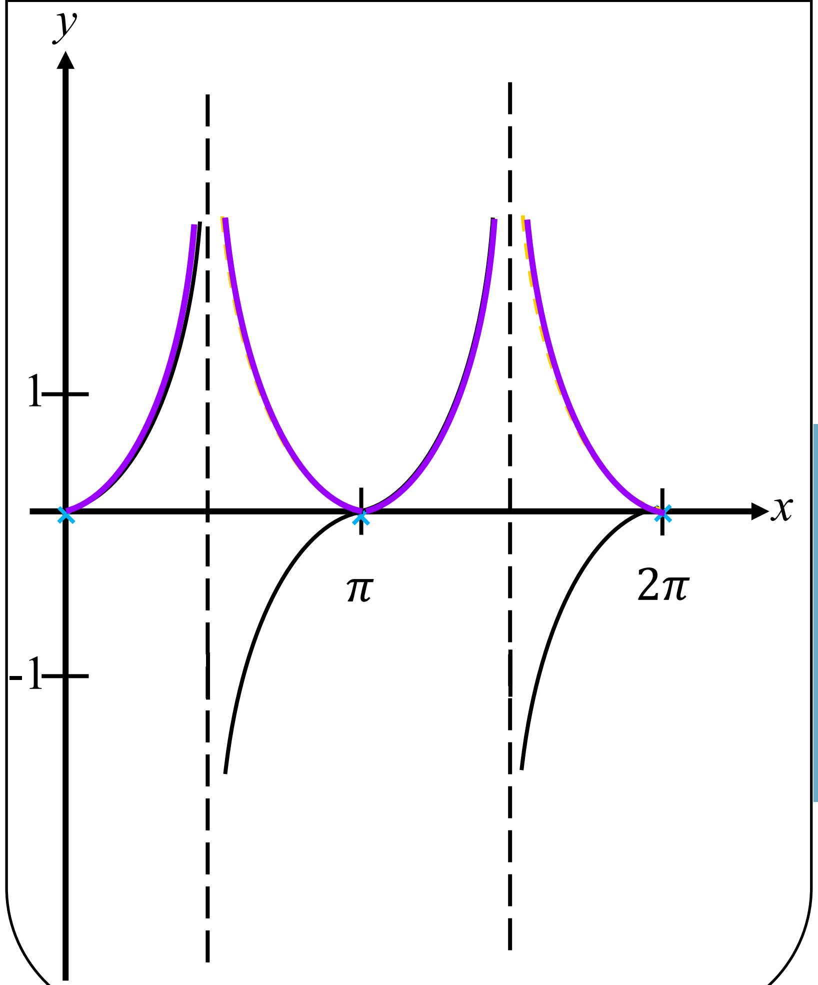
Example 19: $y = -\sin x$



Example 20: $y = |\cos x|$



Example 21: $y = |\tan x|$



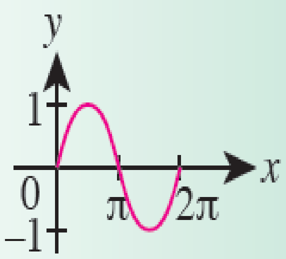
$y = a \sin bx + c$

a

- If $c = 0$:
  Amplitude $= |a|$, Maximum value of $y = a$, Minimum value of $y = -a$
- If $c \neq 0$:
  Amplitude $= |a|$ or $\dfrac{(\text{maximum value} - \text{minimum value})}{2}$

sin

Shape of graph:



b

- Number of cycles in the range $0° \leqslant x \leqslant 360°$ or $0 \leqslant x \leqslant 2\pi$
- Period $= \dfrac{360°}{b}$
  $= \dfrac{2}{b}\pi$

c

Translation $\begin{pmatrix} 0 \\ c \end{pmatrix}$ from the basic graph.

$+\ c$ means moves up c unit

$-\ c$ means moves down c unit

②③① ④

$y = a \sin bx \pm c$
$y = a \cos bx \pm c$
$y = a \tan bx \pm c$

1. Cycle
- Draw intervals
- Number of intervals = **b×4**
- Last interval is always 2π or 360°

2. Amplitude
- Label amplitude on y-axis

3. Trigo. Ratio
- Plot the graph
- If a<0 (negative), plot graph in opposite direction

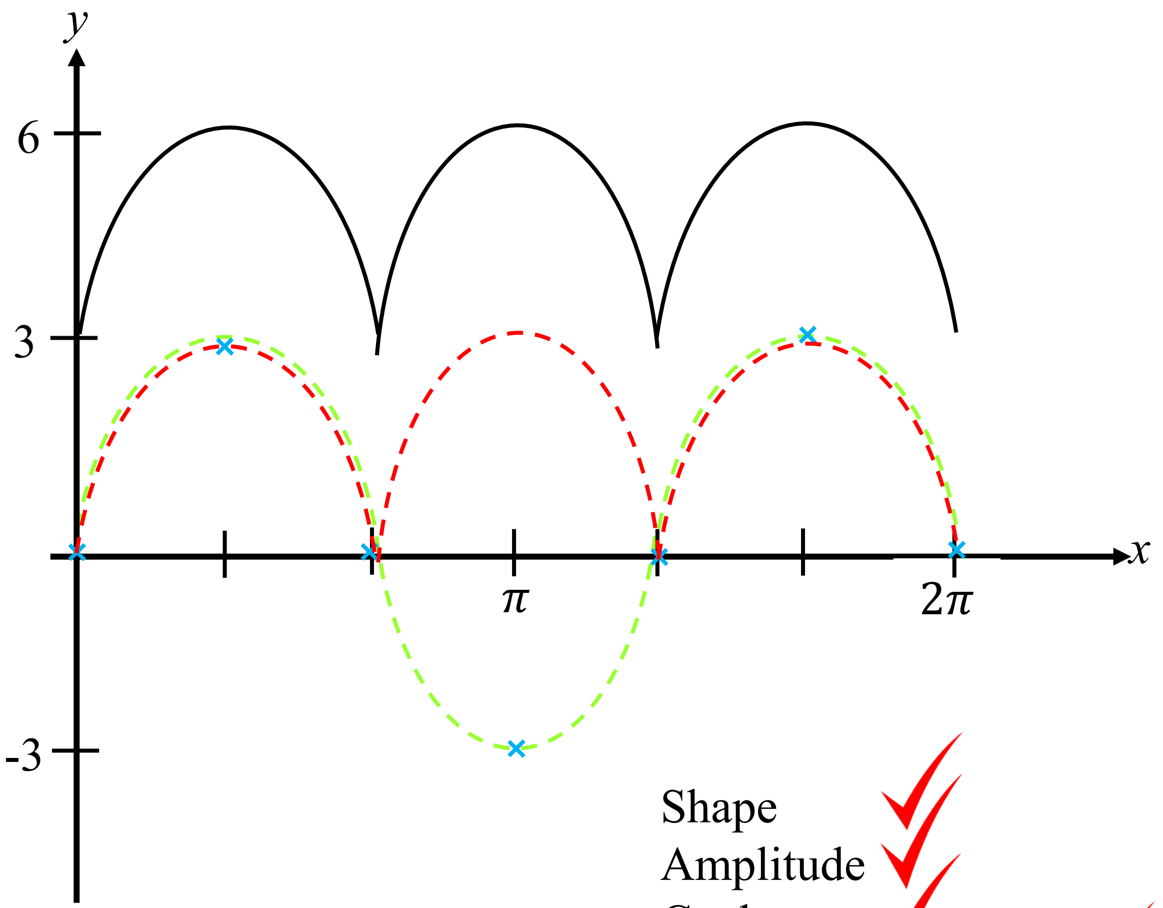
4. Movement
- + C, graph moves up by c unit.
- – C, graph moves down by c unit.

5. Range
- Depends on the question
- Determine where the graph stop

Modulus
- Graph below x-axis are reflected over the x-axis

④②③① ⑤

Example 22: Sketch graph of $y = |\ 3\ \sin\frac{3}{2}x| + 3$ for $0 \leq x \leq 2\pi$



Shape ✓
Amplitude ✓
Cycle ✓
Translation/Absolute ✓

TRIGON. RATIO

② ③ ① ④

$y = a \sin b\,x \pm c$
$y = a \cos b\,x \pm c$
$y = a \tan b\,x \pm c$

1. Cycle
- Draw intervals
- Number of intervals = **b×4**
- Last interval is always 2π or 360°

2. Amplitude
- Label amplitude on y-axis

3. Trigo. Ratio
- Plot the graph
- If a<0 (negative), plot graph in opposite direction

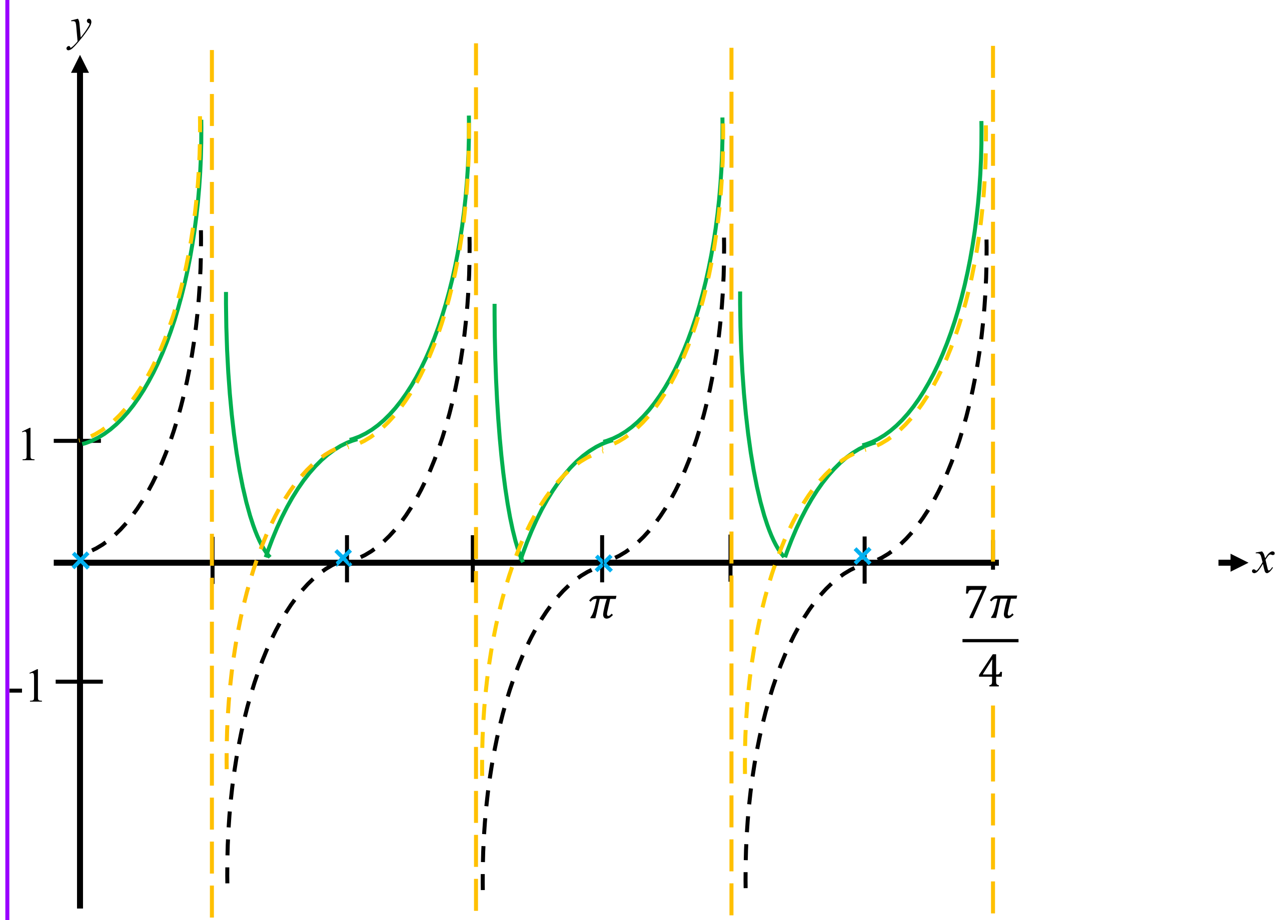
4. Movement
- + C, graph moves up by c unit.
- – C, graph moves down by c unit.

5. Range
- Depends on the question
- Determine where the graph stop

Modulus
- Graph below x-axis are reflected over the x-axis

② ③ ① ④ ⑤

Example 23: Sketch graph of $y = |\ \tan 2x + 1\ |$ for $0 \leq x \leq \frac{7}{4}\pi$



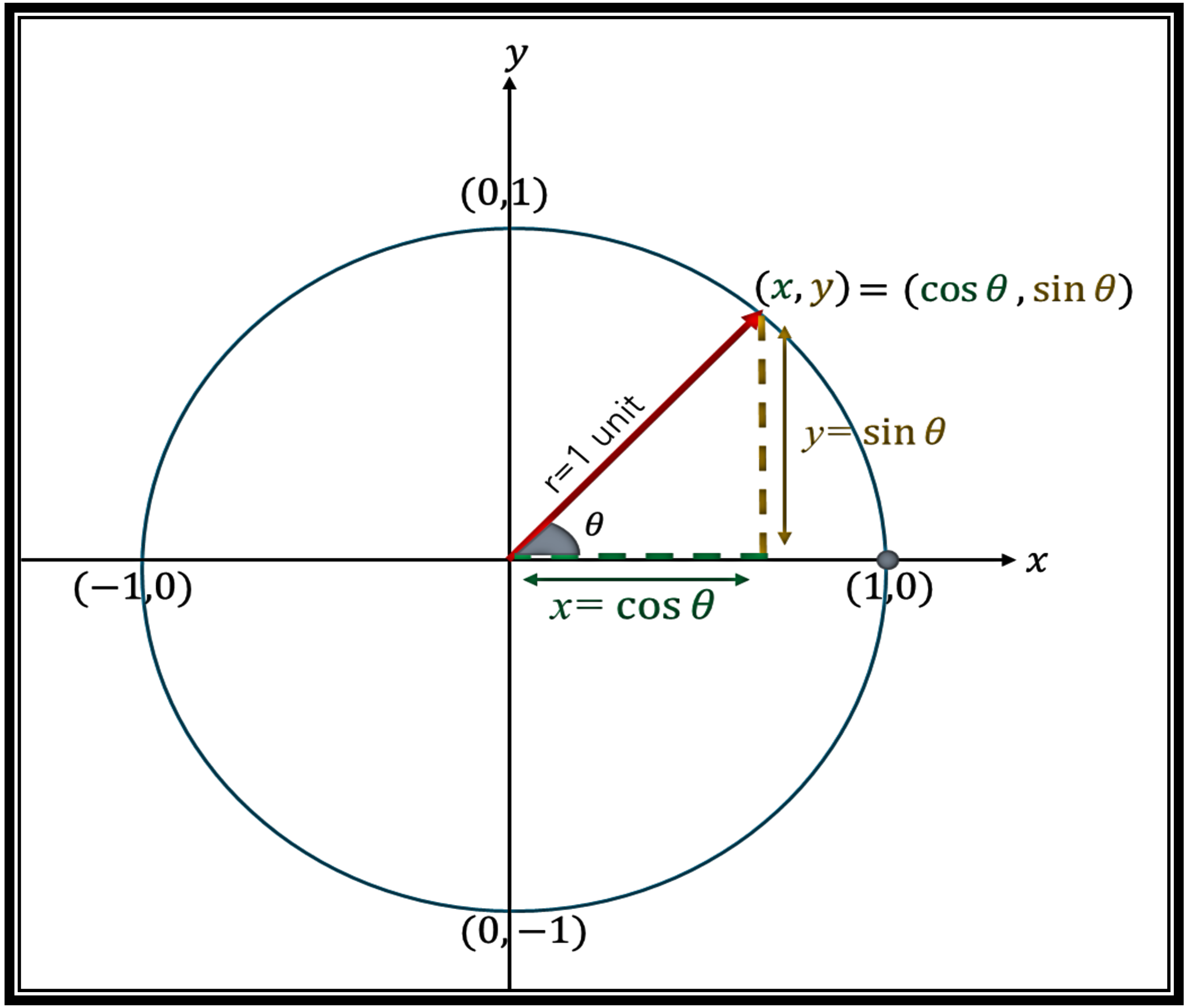
TRIGONOMETRIC FORMULAE

1. $sin^2A + cos^2A = 1$
2. $sec^2A = 1 + tan^2A$
3. $cosec^2A = 1 + cot^2A$
4. $sin(A \pm B) = sin\,A\,cos\,B \pm cos\,A\,sin\,B$
5. $cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$
6. $tan(A \pm B) = \dfrac{tan\,A \pm tan\,B}{1 \mp tan\,A\,tan\,B}$
7. $sin\,2A = 2\,sin\,A\,cos\,A$
8. $cos\,2A = cos^2A - sin^2A$
   $= 2\,cos^2A - 1$
   $= 1 - 2\,sin^2A$
9. $tan\,2A = \dfrac{2\,tan\,A}{1 - tan^2\,A}$

Half-angle formulae:-

10. $sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1 - cos\,A}{2}}$
11. $cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1 + cos\,A}{2}}$
12. $tan\,\dfrac{A}{2} = \dfrac{sin\,A}{1+cos\,A}$

DERIVING THE BASIC IDENTITIES



By using Pythagoras Thm:-

$a^2 + b^2 = c^2$

$sin^2\theta + cos^2\theta = 1$ —— (1)

Let's divide (1) with $sin^2\theta$:-

$$\frac{sin^2\theta}{sin^2\theta} + \frac{cos^2\theta}{sin^2\theta} = \frac{1}{sin^2\theta}$$

$1 + cot^2\theta = cosec^2\theta$ —— (2)

Next, try divide (1) with $cos^2\theta$:-

$$\frac{sin^2\theta}{cos^2\theta} + \frac{cos^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$$

$tan^2\theta + 1 = sec^2\theta$ —— (3)

DERIVING THE DOUBLE ANGLE FORMULAE

Suppose we are given the **Addition Formulae** below;

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin 2A = \sin(A+A) = \sin A \cos A + \cos A \sin A$$
$$= 2 \sin A \cos A$$

$$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A$$
$$= \cos^2 A - \sin^2 A$$

However, by referring to Basic Identities

$$\sin^2 A + \cos^2 A = 1$$

we can further derive $\cos 2A$ as the following

$$= (1 - \sin^2 A) - \sin^2 A$$
$$= 1 - 2\sin^2 A$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1$$

$$\tan 2A = \tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$
$$= \frac{2 \tan A}{1 - \tan^2 A}$$

DERIVING THE HALF ANGLE FORMULAE

$$\sin 2A = 2\sin A\cos A$$

$$\sin\frac{2A}{2} = 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} \quad \text{— (1)}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos\frac{2A}{2} = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$$

$$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} \quad \text{— (2)}$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos\frac{2A}{2} = 1 - 2\sin^2\frac{A}{2}$$

$$\cos A = 1 - 2\sin^2\frac{A}{2} \quad \text{— (3)}$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos\frac{2A}{2} = 2\cos^2\frac{A}{2} - 1$$

$$\cos A = 2\cos^2\frac{A}{2} - 1 \quad \text{— (4)}$$

By rearranging the formula from (3);

$$\cos A = 1 - 2\sin^2\frac{A}{2}$$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

By rearranging the formula from (4);

$$\cos A = 2\cos^2\frac{A}{2} - 1$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

What is the formula for $tan\frac{A}{2}$???

Previously we learned that;

$$sin\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}}$$

$$cos\frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}}$$

Therefore;

$$\tan\frac{A}{2} = \frac{sin\frac{A}{2}}{cos\frac{A}{2}}$$

$$\tan\frac{A}{2} = \frac{\pm\sqrt{\frac{1 - \cos A}{2}}}{\pm\sqrt{\frac{1 + \cos A}{2}}}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{\frac{1 - \cos A}{2}}{\frac{1 + \cos A}{2}}}$$

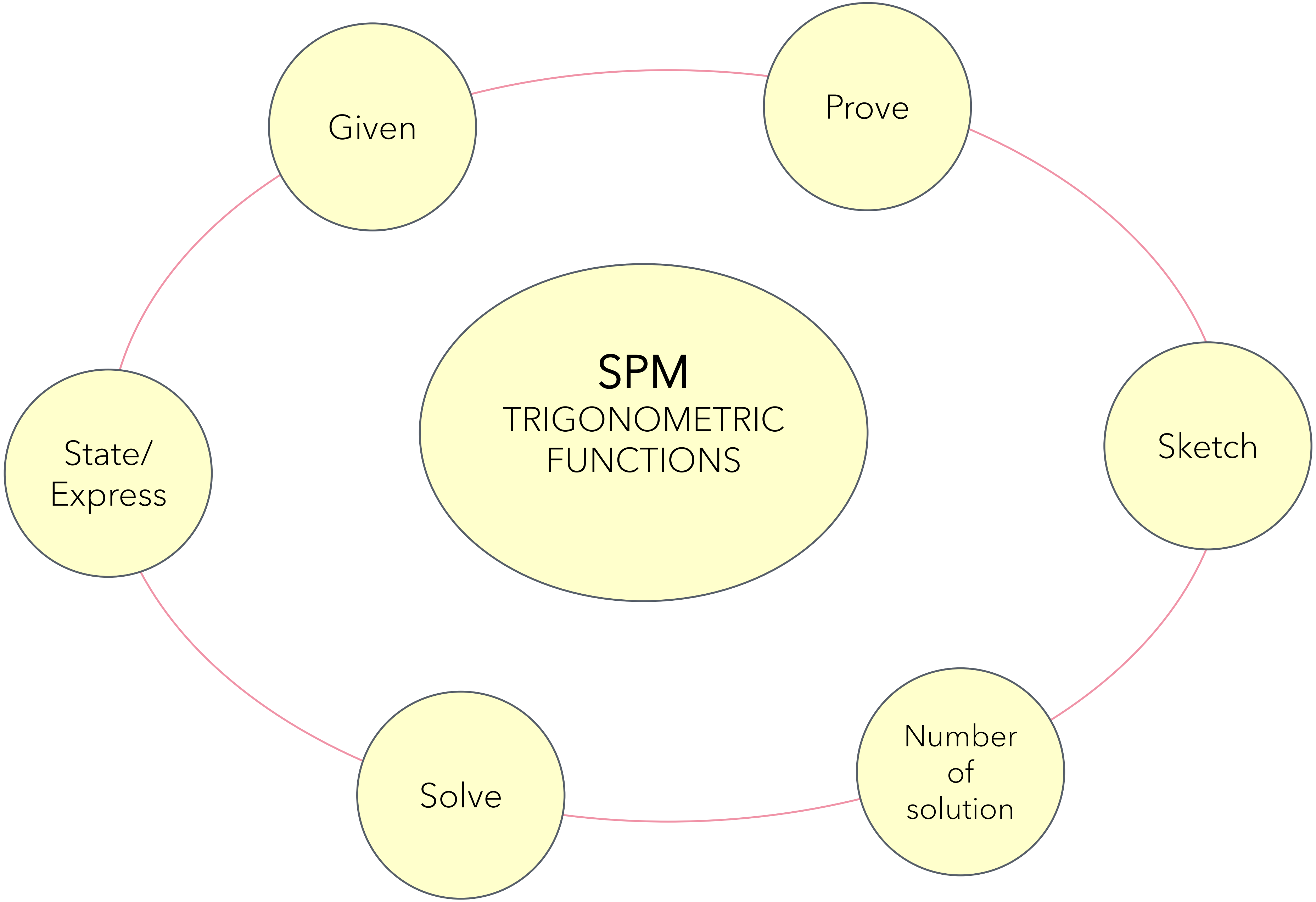
$$\tan\frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(1 - \cos\ A).(1 + \cos A)}{(1 + \cos A).(1 + \cos A)}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{1 - cos^2\ A}{(1 + \cos A)^2}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{sin^2\ A}{(1 + \cos A)^2}}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$


SPM
TRIGONOMETRIC FUNCTIONS
Given
Prove
Sketch
Number of solution
Solve
State/ Express

SPM 2015

Example 24: Prove that $2\cos(x+45°)\cos(x-45°) = cos2x$

$$\text{LHS} = 2\cos(x+45°)\cos(x-45°)$$

$$= 2\,[(\cos x\, cos\, 45 - \sin x \sin 45)(\cos x \cos 45 + \sin x \sin 45)]$$

$$= 2\,[(\cos x\,.\frac{1}{\sqrt{2}} - \sin x\,.\frac{1}{\sqrt{2}})(\cos x\,.\frac{1}{\sqrt{2}} + \sin x\,.\frac{1}{\sqrt{2}})]$$

$$= 2\,[(\cos x\,.\frac{1}{\sqrt{2}})^2 - (\sin x\,.\frac{1}{\sqrt{2}})^2]$$

$$= 2[\frac{cos^2\,x}{2} - \frac{sin^2\,x}{2}]$$

$$= cos^2x - sin^2x$$

$$= cos2x$$

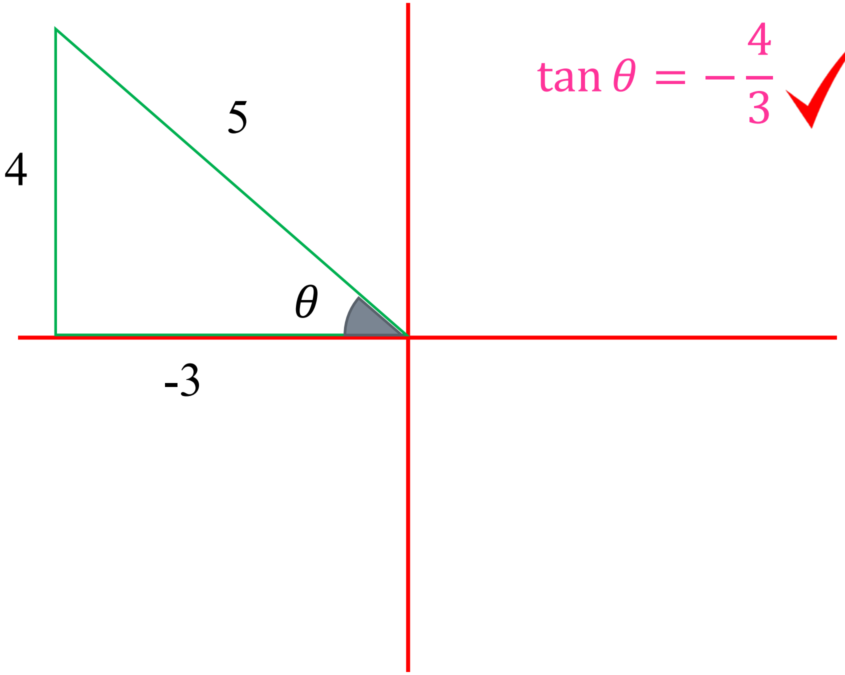
TRIGONOMETRIC FORMULAE

1. $sin^2A + cos^2A = 1$
2. $sec^2A = 1 + tan^2A$
3. $cosec^2A = 1 + cot^2A$
4. $sin(A \pm B) = sin\,A\,cos\,B \pm cos\,A\,sin\,B$
5. $cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$
6. $tan(A \pm B) = \frac{tan\,A \pm tan\,B}{1 \mp tan\,A\,tan\,B}$
7. $sin\,2A = 2\,sin\,A\,cos\,A$
8. $cos\,2A = cos^2A - sin^2A$
   $= 2\,cos^2A - 1$
   $= 1 - 2\,sin^2A$
9. $tan\,2A = \frac{2\,tan\,A}{1 - tan^2\,A}$

SPM 2015

Example 25: Given $\cos\theta = -\frac{3}{5}$ and $0° < \theta < 180°$, find the value of $tan(\theta + 45°)$.

$$\cos\theta = -\frac{3}{5} = \frac{adjacent}{hypotenuse}$$



$$\tan\theta = -\frac{4}{3}$$

$$\tan(\theta + 45°) = \frac{\tan\theta + \tan 45}{1 - \tan\theta\tan 45}$$

$$\tan(\theta + 45°) = \frac{(\frac{-4}{3}) + 1}{1 - (\frac{-4}{3})}$$

$$\tan(\theta + 45°) = \frac{-1}{7}$$

TRIGONOMETRIC FORMULAE

1. $sin^2A + cos^2A = 1$
2. $sec^2A = 1 + tan^2A$
3. $cosec^2A = 1 + cot^2A$
4. $sin(A \pm B) = sin\,A\,cos\,B \pm cos\,A\,sin\,B$
5. $cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$
6. $tan(A \pm B) = \frac{tan\,A \pm tan\,B}{1 \mp tan\,A\,tan\,B}$
7. $sin\,2A = 2\,sin\,A\,cos\,A$
8. $cos\,2A = cos^2A - sin^2A$ $= 2\,cos^2A - 1$ $= 1 - 2\,sin^2A$
9. $tan\,2A = \frac{2\,tan\,A}{1 - tan^2\,A}$

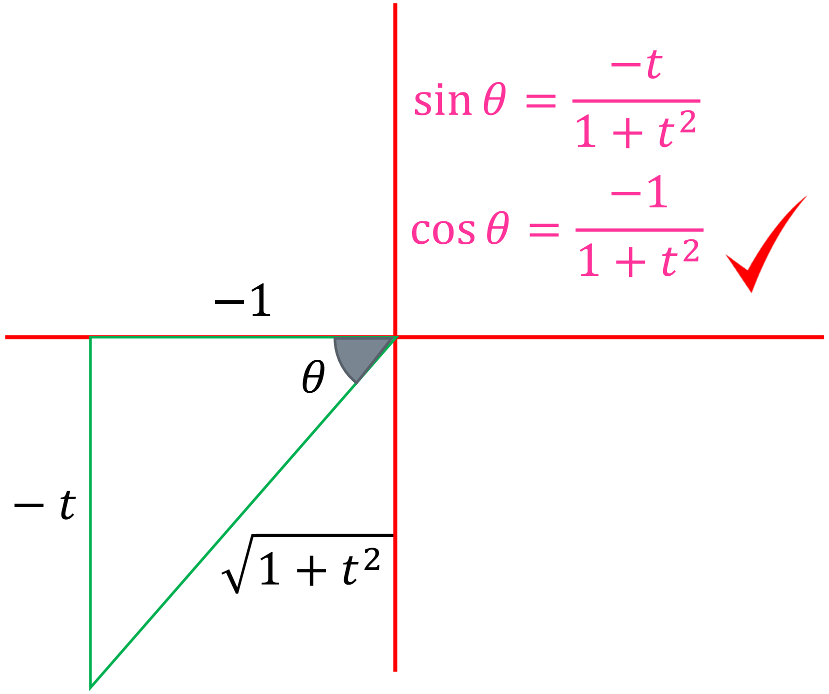
TRIGON. RATIO

SPM CLONE

Example 26: It is **given** that $\tan\theta = t$ where θ is a reflex angle. Express in terms of t

a) $cosec\ 2\theta$

$$\tan\theta = \frac{t}{1} = \frac{opposite}{adjacent}$$



$$\sin\theta = \frac{-t}{1+t^2}$$

$$\cos\theta = \frac{-1}{1+t^2}$$

$$cosec\ 2\theta = \frac{1}{sin\ 2\theta}$$

$$cosec\ 2\theta = \frac{1}{2\sin\theta\cos\theta}$$

$$= \frac{1}{2\left(\frac{-t}{\sqrt{1+t^2}}\right)\left(\frac{-1}{\sqrt{1+t^2}}\right)}$$

$$= \frac{1}{\left(\frac{2t}{\sqrt{(1+t^2)}^2}\right)}$$

$$= \frac{(1+t^2)}{2t}$$

TRIGONOMETRIC FORMULAE

1. $sin^2A + cos^2A = 1$
2. $sec^2A = 1 + tan^2A$
3. $cosec^2A = 1 + cot^2A$
4. $sin(A \pm B) = sin\ A\ cos\ B \pm cos\ A\ sin\ B$
5. $cos(A \pm B) = cos\ A\ cos\ B \mp sin\ A\ sin\ B$
6. $tan(A \pm B) = \dfrac{tan\ A \pm tan\ B}{1 \mp tan\ A\ tan\ B}$
7. $sin\ 2A = 2\ sin\ A\ cos\ A$
8. $cos\ 2A = cos^2A - sin^2A$
   $= 2\ cos^2A - 1$
   $= 1 - 2\ sin^2A$
9. $tan\ 2A = \dfrac{2\ tan\ A}{1 - tan^2\ A}$

TRIGON. RATIO

SPM CLONE

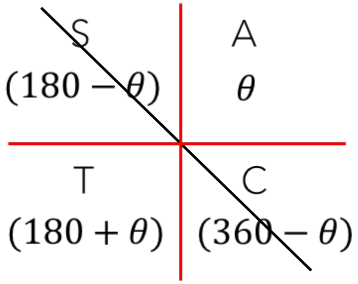
Example 27: Solve the equation $sin\ 2x + cos\ 2x = 0$ for $0° \leq x \leq 360°$

$$sin\ 2x + cos\ 2x = 0$$

$$\frac{sin\ 2x}{cos\ 2x} + \frac{cos\ 2x}{cos\ 2x} = 0$$

$$tan\ 2x + 1 = 0$$

$$tan\ 2x = -1$$



$$2(0°) \leq 2x \leq 2(360°)$$

$$0° \leq 2x \leq 720°$$

1) Quadrant
2) Range
3) Reference Angle
4) Corresponding Angle

$$\alpha = tan^{-1}1$$

$$\alpha = 45°$$

$$2x = (180° - 45°), (360° - 45°), (135° + 360°), (315° + 360°)$$

$$x = \frac{135°}{2}, \frac{315°}{2}, \frac{495°}{2}, \frac{675°}{2}$$

$$x = 67.5°, 157.5°, 247.5°, 337.5°$$

SPM CLONE

Example 28: Solve the equation $sin\ 2x - 2\ sin\ x = 0$ for $0° \leq x \leq 360°$

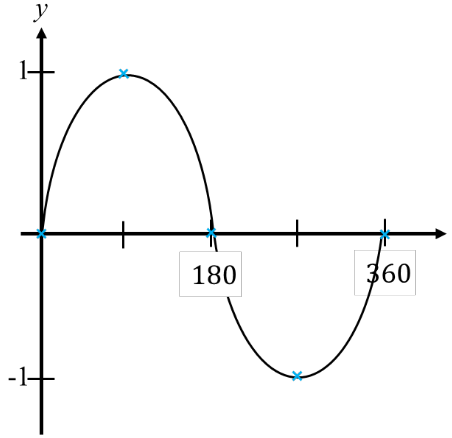
$sin\ 2x - 2\ sin\ x = 0$

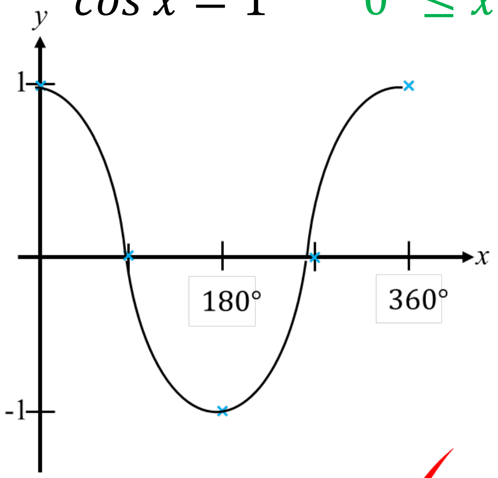
$2\ sin\ x \cos x - 2\ sin\ x = 0$

$2\ sin\ x\ (cos\ x - 1) = 0$

$2\ sin\ x = 0$ $\qquad$ $cos\ x - 1 = 0$

$sin\ x = 0$ $\qquad$ $cos\ x = 1$ $\qquad$ $0° \leq x \leq 360°$





$x = 0°, 180°, 360°$ $\qquad$ $x = 0°, 360°$

$x = 0°, 180°, 360°$

TRIGONOMETRIC FORMULAE

1. $sin^2 A + cos^2 A = 1$
2. $sec^2 A = 1 + tan^2 A$
3. $cosec^2 A = 1 + cot^2 A$
4. $sin(A \pm B) = sin\ A\ cos\ B \pm cos\ A\ sin\ B$
5. $cos(A \pm B) = cos\ A\ cos\ B \mp sin\ A\ sin\ B$
6. $tan(A \pm B) = \dfrac{tan\ A \pm tan\ B}{1 \mp tan\ A\ tan\ B}$
7. $sin\ 2A = 2\ sin\ A\ cos\ A$
8. $cos\ 2A = cos^2 A - sin^2 A$
   $= 2\ cos^2 A - 1$
   $= 1 - 2\ sin^2 A$
9. $tan\ 2A = \dfrac{2\ tan\ A}{1 - tan^2\ A}$

TRIGON. RATIO

SPM 2016

Example 29: Solve the equation $tan\ \theta = 4 - 3\ cot\ \theta$ for $0° \leq \theta \leq 180°$

$$tan\ \theta = 4 - 3\ cot\ \theta$$

$$\left[tan\ \theta = 4 - 3\frac{1}{tan\ \theta}\right](tan\ \theta)$$

$$tan^2\ \theta = 4\ tan\ \theta - 3$$

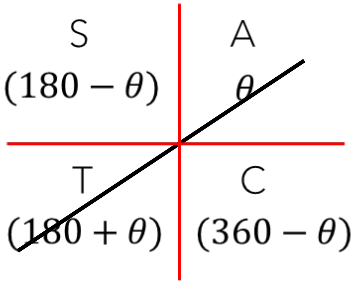
$$tan^2\ \theta - 4\ tan\ \theta + 3 = 0$$

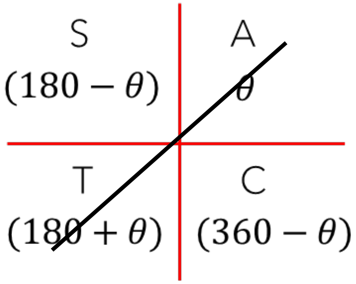
$$(tan\ \theta - 3)(tan\ \theta - 1) = 0$$

1) Quadrant
2) Range
3) Reference Angle
4) Corresponding Angle

trivago

$tan\ \theta = 3$ $\qquad$ $tan\ \theta = 1$ $\qquad$ $0° \leq \theta \leq 180°$





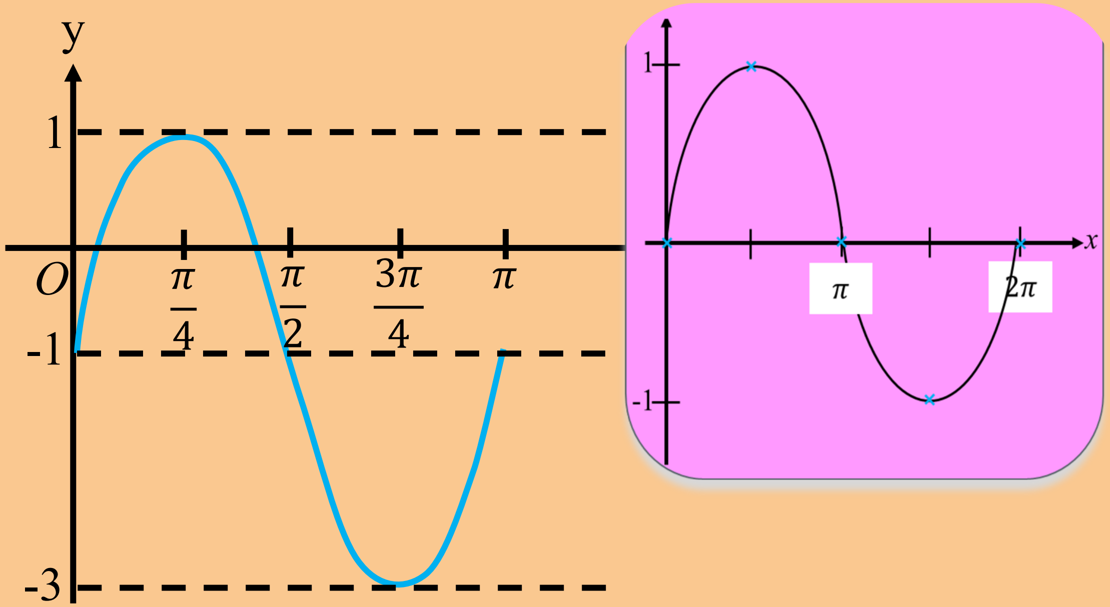
$\theta = tan^{-1}3$ $\qquad$ $\theta = tan^{-1}1$

$\theta = 71.57°$ $\qquad$ $\theta = 45°$

$$\theta = 45°, 71.57°$$

SPM CLONE

Example 30: Diagram shows part of the graph $y = a \sin bx + c$ for $0 \le x \le \pi$



State the values of a, b and c.

$a = amplitude = \frac{max - min}{2}$

$a = \frac{1-(-3)}{2}$

$a = 2$ (positive direction)

$b =$ number of cycles from 0 to 2π

$period = \frac{2\pi}{b}$

$\pi = \frac{2\pi}{b}$

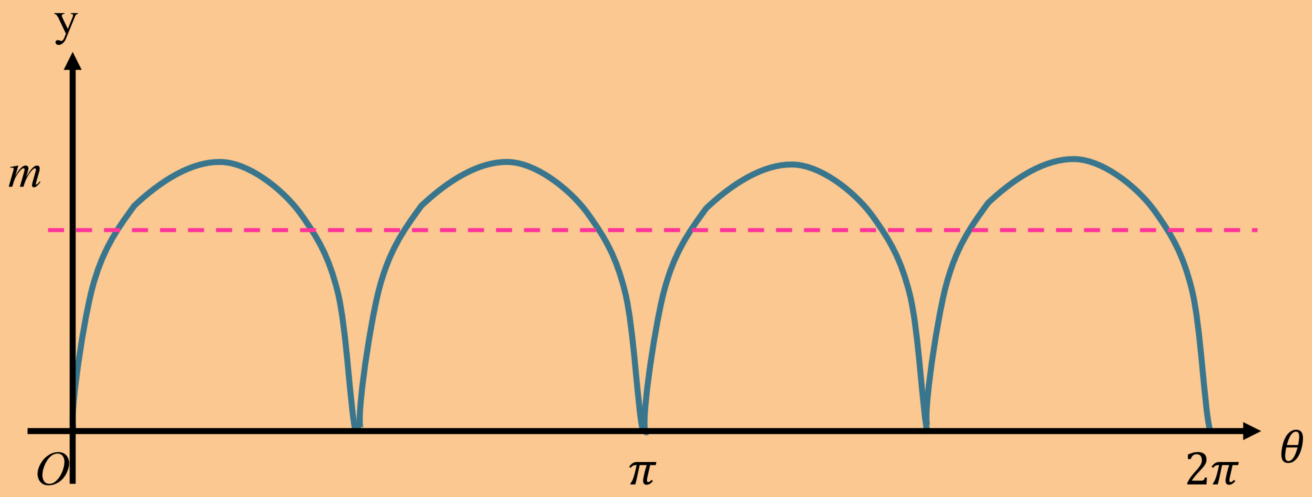
$\therefore b = 2$

$c =$ vertical translation

$c = -1$ (moving downward by 1 unit)

SPM 2019

Example 31: Diagram shows the graph of $y = |n \sin\theta \cos\theta|$ for $0 \le \theta \le 2\pi$



a) Express m in terms of n.

b) There are 8 solutions when $y = k$, where k is a constant. State the range of k in terms of m.

$$y = |n \sin\theta \cos\theta|$$

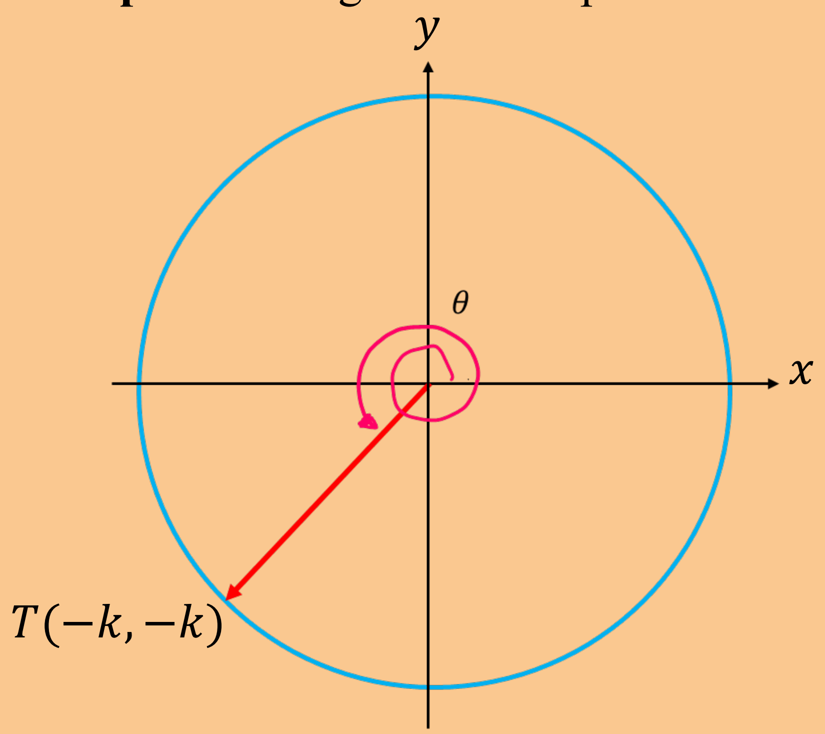
$$|\sin 2\theta| = |2 \sin\theta \cos\theta|$$

$$n = 2$$
$$m = 1$$
$$\therefore m = \frac{n}{2}$$ ✓

$$0 < k < m$$ ✓

SPM CLONE

Example 32: Diagram shows point T on a unit circle.



a) State the value of θ.

b) Express $2\cos(-\theta)$ in terms of k.

$$\theta = 540^\circ + 45^\circ$$
$$\theta = 585^\circ$$

$$\cos(-\theta) = -k$$

$$2\cos(-\theta) = 2(-k)$$
$$= -2k$$

SPM 2016

Example 33:

a) Prove $\frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x} = \tan 2x$

$$\text{LHS} = \frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}$$

$$= \frac{\sin 2x}{\tan^2 x + 2\cos^2 x - (1 + \tan^2 x)}$$

$$= \frac{\sin 2x}{\tan^2 x + 2\cos^2 x - 1 - \tan^2 x}$$

$$= \frac{\sin 2x}{2\cos^2 x - 1}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

TRIGONOMETRIC FORMULAE

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A = 1 + \tan^2 A$
3. $\operatorname{cosec}^2 A = 1 + \cot^2 A$
4. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
5. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
6. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
7. $\sin 2A = 2 \sin A \cos A$
8. $\cos 2A = \cos^2 A - \sin^2 A$
   $= 2\cos^2 A - 1$
   $= 1 - 2\sin^2 A$
9. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

SPM 2016

Example 33:

b) Sketch the graph of $y = |\tan 2x|$ for $0 \le x \le 2\pi$

c) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\left|\frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}\right| + \frac{x}{2\pi} = 1$ for $0 \le x \le 2\pi$

$y = |\tan 2x|$ —— (1)

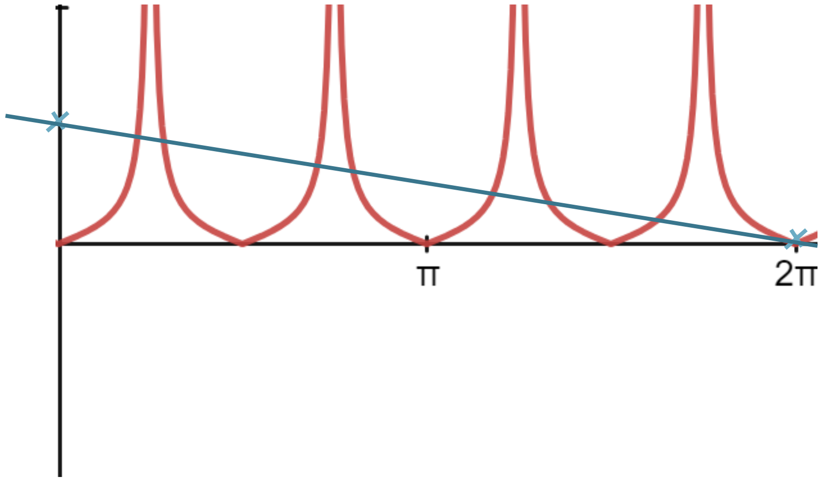
$$\left|\frac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}\right| = |\tan 2x|$$

$|\tan 2x| + \frac{x}{2\pi} = 1$ —— (2)

$$y + \frac{x}{2\pi} = 1$$

$$y = 1 - \frac{x}{2\pi}$$

x	0	2π
y	1	0



8 number of solutions

SPM CLONE

Example 34:

a) Sketch the graph of $y = -\frac{1}{2}\sin A$ for $0 \le A \le 2\pi$

b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $(\tan A)\left(2\cos^2\frac{A}{2} - 1\right) = \frac{A}{2\pi}$ for $0 \le A \le 2\pi$

State the number of solutions.

$$y = -\frac{1}{2}\sin A \quad \text{--- (1)}$$

$$(\tan A)\left(2\cos^2\frac{A}{2} - 1\right) = \left(\frac{\sin A}{\cos A}\right)\cos A$$

$$= \sin A$$

$$\sin A = \frac{A}{2\pi} \quad \text{--- (2)}$$

$$y = -\frac{1}{2}\left(\frac{A}{2\pi}\right)$$

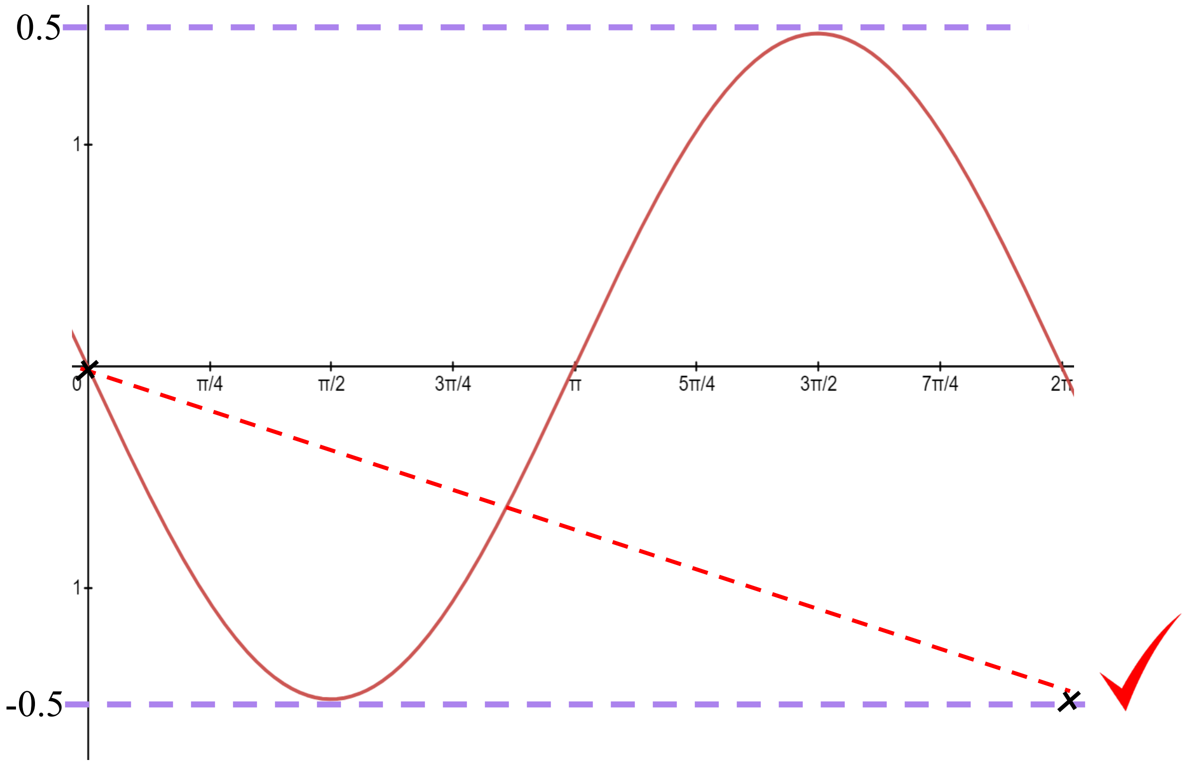
$$y = -\frac{A}{4\pi}$$ ✓

A	0	2π
y	0	-0.5

TRIGONOMETRIC FORMULAE

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A = 1 + \tan^2 A$
3. $\mathrm{cosec}^2 A = 1 + \cot^2 A$
4. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
5. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
6. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
7. $\sin 2A = 2\sin A \cos A$
8. $\cos 2A = \cos^2 A - \sin^2 A$
   $= 2\cos^2 A - 1$
   $= 1 - 2\sin^2 A$
9. $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

A	0	2π
y	0	-0.5



2 number of solutions

Siri Jom Skor A+ Matematik Tambahan

SPM 2021

22 Aug
8.00 pm – 10.00 pm
Sahlawati Zakaria | MRSM Kuala Krai
Functions

27 Aug
8.00 pm – 10.00 pm
Norlela Sapari | MRSM Taiping
Quadratic Functions

31 Aug
8.00 pm – 10.00 pm
Khairulbariah Khairuddin | MRSM Mersing
Systems of Equations

4 Sept
8.00 pm – 10.00 pm
Hazlina Ahmad | MRSM Alor Gajah
Indices, Surds and Logarithms

10 Sept
8.00 pm – 10.00 pm
Hasniza Ismail | MRSM Parit
Progressions

16 Sept
3.00 pm – 5.00 pm
Rosdiana Sarju | MRSM Johor Bahru
Linear Law

24 Sept
8.00 pm – 10.00 pm
Nur Suhaila Abu Bakar | MRSM Tumpat
Coordinate Geometry
New Speaker

26 Sept
8.00 pm – 10.00 pm
Mohd Faizi Mamat | MRSM Gemencheh
Vectors

1 Oct
8.00 pm – 10.00 pm
Abdul Hadi Azmi | MRSM Pengkalan Chepa
Solution of Triangles

8 Oct
8.00 pm – 10.00 pm
Noraini Ismail | MRSM Transkrian
Index Numbers

10 Oct
8.00 pm – 10.00 pm
Hariani Abidin | MRSM Kuching
Circular Measure

15 Oct
8.00 pm – 10.00 pm
Erwan Hazreen Musa | MRSM Bentong
Differentiation

18 Oct
8.00 pm – 10.00 pm
Mohamad Fauzi Razak | MRSM Kepala Batas
Integration
New Date

23 Oct
8.00 pm – 10.00 pm
Muhamad Baginda Zainuddin | MRSM Batu Pahat
Kinematics of Linear Motion
New Date

30 Oct
3.00 pm – 5.00 pm
Haziq Syazwan Sajali | MRSM Tun Mustapha
Trigonometric Function
New Date

5 Nov
3.00 pm – 5.00 pm
Suhaila Sulong | MRSM Tun Dr. Ismail
Permutation and Combination
New Date

7 Nov
8.00 pm – 10.00 pm
Norhafizah Mohamed Yusoff | MRSM K. Terengganu
Probability Distribution
New Date

17 Dis
8.00 pm – 10.00 pm
Asniza Arshad | MRSM Tun Ghaffar Baba
Linear Programming

Anjuran Unit Matematik
Bahagian Pendidikan Menengah MARA

Sesi webinar *live* melalui Microsoft Teams